

# Non-CES Models of Monopolistic Competition

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ECON245 - Winter 24

# Modeling market power

- ▶ Workhorse models of heterogeneous firms feature
  1. CES demand
  2. Monopolistic competition
- ▶ non-starter: markups are (i) homogeneous and (ii) exogenous

$$p_{it} = \frac{\sigma}{\sigma - 1} mc_{it}$$

- ▶ two (immediate) ways out
  1. maintain CES demand, assume oligopoly
  2. non-CES demand, maintain monopolistic competition

# Special Features of CES

- ▶ income elasticity of each good is one (homotheticity)
- ▶ relative demand for any two goods depends only on the ratio of prices of these two goods (independence of irrelevant alternatives)
- ▶ EoS btw all pairs of goods are not only constant but also identical
- ▶ goods are either gross complements (expenditure share is increasing in the relative price if  $\sigma < 1$ ) or gross substitutes (if  $\sigma > 1$ )
- ▶ either all goods are essential ( $p_\omega \rightarrow \infty \Rightarrow P \rightarrow \infty$ , if  $\sigma \leq 1$ ) or inessential
- ▶ no choke prices: demand for any good remains strictly positive at all relative prices, no saturation

# From Matsuyama 2023

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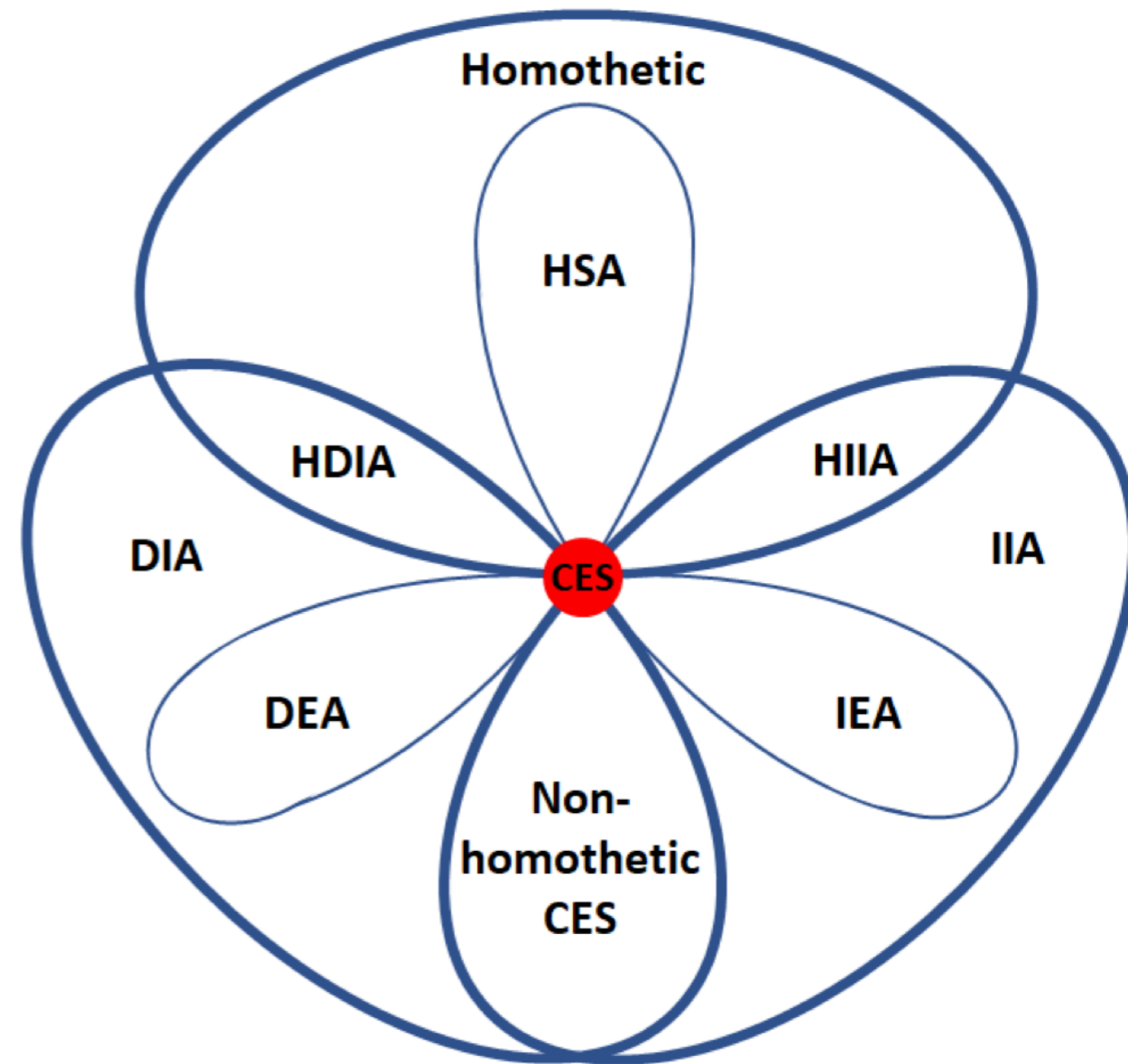
## Landscape of non-CES Aggregators

$DEA \cap \text{Homothetic} = \text{CES}$   
Known as “Bergson’s Law”

$DEA \cap \text{IEA} = \text{CES}$ .  
Samuelson (1965)

$\text{IEA} \cap \text{Homothetic} = \text{CES}$ .  
Berndt and Christensen (1973)

Departing from CES in the direction of DEA or IEA introduces nonhomotheticity.



# DEA

$$U(\mathbf{c}) = \sum_{\Omega} u_{\omega}(c_{\omega})$$

▶ non-nomothetic unless CES, i.e.,  $u_{\omega}(x) \equiv a_{\omega}x^{(\sigma-1)/\sigma}$

▶ demand:

$$c_{\omega} = D_{\omega}(p_{\omega}/A), D_{\omega}(\cdot) \equiv (u'_{\omega})^{-1}(\cdot), A \equiv [\sum_{\Omega} u'_{\omega}(c_{\omega})c_{\omega}d\omega]/E$$

▶ flexible demand elasticity:  $\sigma_{\omega}(p/A) \equiv -\frac{\partial \ln D_{\omega}(p/A)}{\partial \ln(p/A)}$

▶ but: MRS btw  $\omega, \omega'$  is independent of any other good:

$$p_{\omega}/p_{\omega'} = u'_{\omega}(c_{\omega})/u'_{\omega'}(c_{\omega'})$$

▶ so variable markups stem from non-homotheticity, not from competition

# DEA: Examples

$$U(\mathbf{c}) = \sum_{\Omega} u_{\omega}(c_{\omega})$$

▶ quasi linear:  $U(\mathbf{c}) = C_o + \int_{\Omega} u_{\omega}(c_{\omega})d\omega$

▶ Stone-Geary:  $U(\mathbf{c}) = \sum_{i=1}^N \beta_i \frac{(x_i - \bar{x}_i)^{(\sigma-1)/\sigma}}{1 - 1/\sigma}$

- $\bar{x}_i$  is the subsistence level, ladder of development in consumption: relative expenditures on  $i$  and  $j$  is decreasing in  $E \Leftrightarrow \bar{x}_i > \bar{x}_j$
  - but: relative MPC is constant in  $E$
  - asymptotically homothetic, i.e., as income goes to infinity, MRS between any two goods becomes independent of income
- ▶ also: Houthakker's addilog (1960), Caron et al (2014) constant ratio of income elasticities

# Non-homothetic CES

- ▶ by far the most popular function to model non-homotheticity these days: Bohr et al (2021), Comin et al (2021), Cravino-Sotelo 2019, Fujiwara-Matsuyama (2022), Matsuyama (2019), ...

- ▶ direct utility is implicitly defined

$$\left[ \sum_{i=1}^n \beta_i(U)^{1/\sigma(U)} \left( \frac{c_i}{U} \right)^{1-\frac{1}{\sigma(U)}} \right]^{\sigma(U)/(\sigma(U)-1)} = 1$$

- ▶ price index  $P = P(\mathbf{p}, U)$  satisfies  $PU = E$

$$\left[ \sum_{i=1}^N \frac{\beta_i(U)}{U^{1-1/\sigma(U)}} \left( \frac{p_i}{P} \right)^{1-\sigma(U)} \right]^{1/(1-\sigma(U))} = 1$$

- ▶ non-homothetic if  $\partial\beta_i(U)/\partial U$  depends on  $i$  or  $\sigma(U)$  depends on  $U$

# Isoelastic non-nomothetic CES

- ▶ set  $\beta_i(U) = \beta_i \times U^{\varepsilon_i - \sigma}$ . and  $\sigma(U) = \sigma$
- ▶ require  $(\varepsilon_i - \sigma)/(1 - \sigma) > 0$  to be well-behaved
- ▶ budget shares:

$$m_i = p_i c_i / E = \beta_i \left( \frac{E}{P} \right)^{\varepsilon_i - 1} \left( \frac{p_i}{P} \right)^{1 - \sigma}$$

- ▶ relative expenditure shares are log-linear in  $E/P = U$  and  $p_i$

$$\ln \left( \frac{m_i}{m_j} \right) = \ln \frac{\beta_i}{\beta_j} + (1 - \sigma) \ln \frac{p_i}{p_j} + (\varepsilon_i - \varepsilon_j) \ln \frac{E}{P}$$

- ▶ advantages over Stone-Geary: not nomothetic in the limit, log-additive expenditure shares (nice for estimation)



# Homothetic non-CES

- ▶ non-homotheticity is useful in some contexts: structural transformation, development, inequality
- ▶ in many (most) other contexts, not giving up homotheticity is important
  - CRS is key for aggregation in macro
  - in nested structures (multi-industry, also multi-region), having non-homotheticities anywhere but at “top” creates problems
- ▶ in markup/market power context, effect of non-homotheticities is often hard to separate from the effects of concentration/competition (or sometimes, confused with it see DEA)

# Homothetic demand systems

- ▶ if  $U(\mathbf{c})$  is homothetic, then it has a dual representation in the form of an ideal price index, defined as  $P = P(\mathbf{p}) = \min_{\mathbf{c}} \{\mathbf{p}\mathbf{c} \mid U(\mathbf{c}) \geq 1\}$
- ▶  $P = P(\mathbf{p})$  is also often called the unit cost function
- ▶ homotheticity:  $P(\mathbf{p})$  indep. of  $U$ , linear homogeneity:  $P(\lambda\mathbf{p}) = \lambda P(\mathbf{p})$
- ▶ demand solves:  $\mathbf{c}(\mathbf{p}) = \arg \min_{\mathbf{c}} \{\mathbf{p}\mathbf{c} \mid U(\mathbf{c}) \geq U\}$
- ▶ Shephard's lemma I:  $\partial E / \partial U = P(\mathbf{p})$
- ▶ Shephard's lemma II:  $c_{\omega}(\mathbf{p}) = \partial E / \partial p_{\omega} = \frac{\partial P(\mathbf{p})}{\partial p_{\omega}} U$

# Homothetic demand systems

- ▶ re-arrange  $c_\omega(\mathbf{p}) = \partial E / \partial p_\omega = \frac{\partial P(\mathbf{p})}{\partial p_\omega} U$  and  $P(\mathbf{p})U = E$  to obtain

$$m_\omega = \frac{p_\omega c_\omega}{E} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega}$$

- ▶ Euler's theorem implies that market shares add to 1

$$\sum_{\Omega} m_\omega = \sum_{\Omega} \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = 1, \text{ or } \int_{\Omega} m_\omega d\omega = \int_{\omega} \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} d\omega = 1$$

- ▶ hence, *any* homothetic demand system is fully characterized these two equations  $P(\mathbf{p})$

- ▶ the CES case is very special: the second equation directly defines  $P$

$$m_\omega = a_\omega \left( \frac{p_\omega}{P} \right)^{1-\sigma}, a_\omega > 0 \text{ and } \sum_{\Omega} m_\omega = 1 \Leftrightarrow P^{1-\sigma} = \sum_{\Omega} a_\omega p_\omega^{1-\sigma}$$

# Homothetic with a Single Aggregator (HSA)

- ▶ HSA: the market share function *directly* becomes the primitive
- ▶ a homothetic  $U(\mathbf{c})$  is called HSA if a good's market shares  $m_\omega$  depends only on its price relative to a common aggregator  $A(\mathbf{p})$  such that

$$m_i = \frac{p_i c_i}{E} = s_\omega \left( \frac{p_\omega}{A(\mathbf{p})} \right)$$

$$\int_{\Omega} s_\omega \left( \frac{p_\omega}{A(\mathbf{p})} \right) = 1$$

- ▶  $s_\omega(\cdot)$  are the primitives, assumptions:  $s'_\omega < 0$  (gross substitutes),  $\lim_{z \rightarrow \bar{z}} s_\omega(z) = 0$  (if  $\bar{z}$  is finite, this is the choke price)
- ▶  $A(\mathbf{p})$  is a price aggregator: mediates competition/cross-price effects in the demand system
- ▶ HSA is new (2017): not much usage yet (see Kimball later), but in my view the most tractable option

# Link between $P$ and $A$

- ▶  $A(\mathbf{p})$  captures cross-price in the demand system

$$\int_{\Omega} s_{\omega} \left( \frac{p_{\omega}}{A(\mathbf{p})} \right) = 1$$

- ▶  $P(\mathbf{p})$  captures welfare effects of prices
- ▶  $A(\mathbf{p})$  is related to the ideal price  $P(\mathbf{p})$  index via (M&U 2017 for proof)

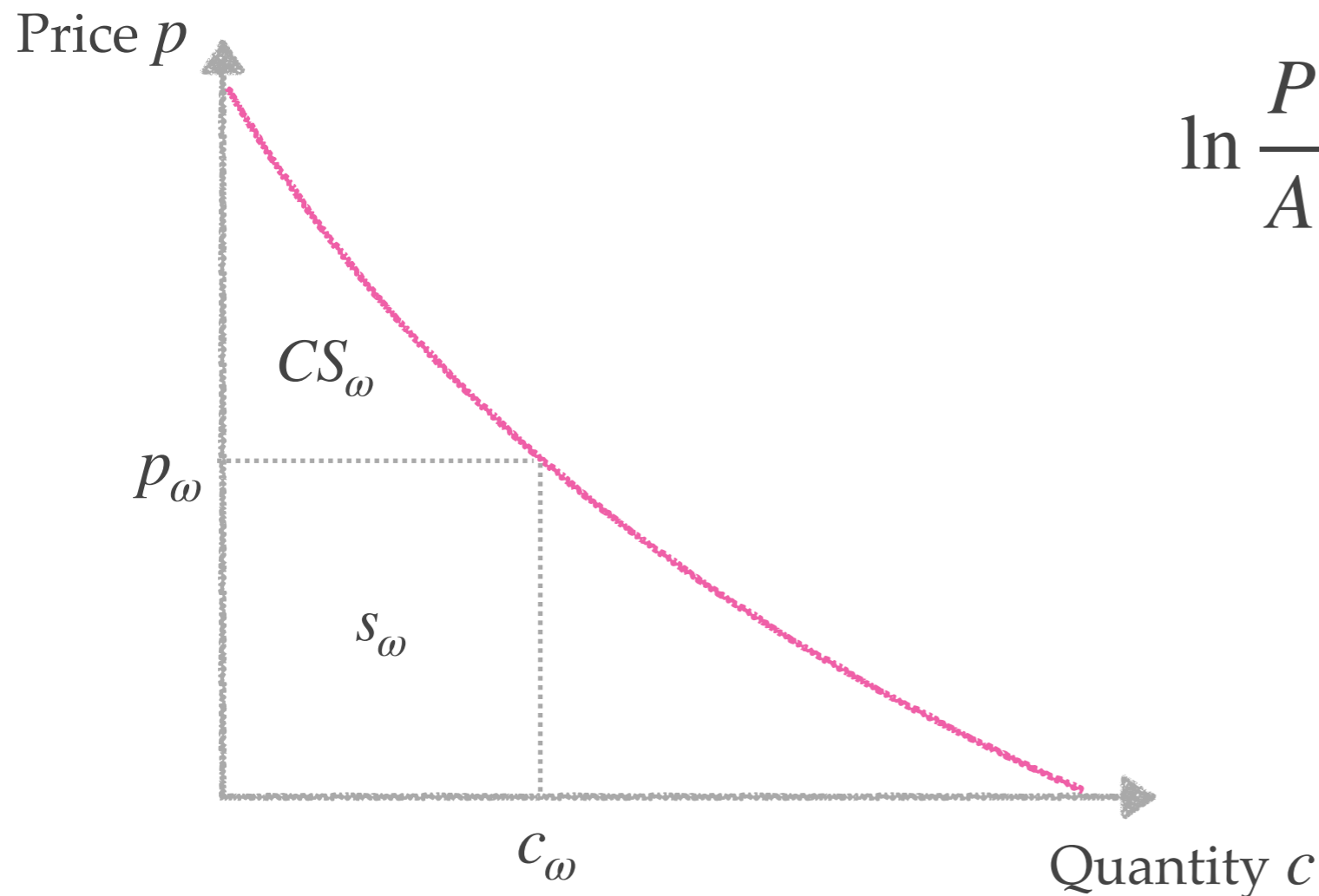
$$\ln \frac{P(\mathbf{p})}{A(\mathbf{p})} = c_1 - \int_{\Omega} \left[ \int_{p_{\omega}/P}^{\infty} \frac{s_{\omega}(\zeta)}{\zeta} d\zeta \right] d\omega$$

- ▶  $c_1$  is an inconsequential constant (depends on normalization)
- ▶ the difference between  $P$  and  $A$  is consumer surplus: the value of having varieties available at a given relative price vector  $\mathbf{p}/P$

# Consumer surplus

$$\delta_{\omega}\left(\frac{P_{\omega}}{A}\right) \equiv 1 + \int_{P_{\omega}/P}^{\infty} \frac{s_{\omega}(\zeta)}{\zeta} d\zeta = \frac{\int_{P_{\omega}}^{\infty} c_{\omega}(p) dp}{s_{\omega}\left(\frac{P_{\omega}}{P}\right)} \geq 1$$

- ▶ hence, can express the difference between  $A$  and  $P$  as a sales-weighted average of the ratios of consumer surplus to sales across all goods



$$\ln \frac{P(\mathbf{p})}{A(\mathbf{p})} = c_1 + \int_{\Omega} s_{\omega} \left[ \frac{CS_{\omega}}{s_{\omega}} \right] d\omega$$

# CES as a special case

- ▶ HSA is CES if  $\forall \omega, s_{\omega}(z) = a_{\omega}z^{1-\sigma}$ ,  $A(\mathbf{p})$  captures cross-price effects in demand:

$$\int_{\Omega} a_{\omega} \left( \frac{p_{\omega}}{A(\mathbf{p})} \right)^{1-\sigma} d\omega = 1 \Leftrightarrow A(\mathbf{p})^{1-\sigma} = \int_{\Omega} a_{\omega} p_{\omega}^{1-\sigma} d\omega$$

- ▶  $A(\mathbf{p})$  comoves one-to-one with the ideal price index  $P(\mathbf{p})$

$$\frac{P(\mathbf{p})}{A(\mathbf{p})} = c_1 \times \frac{1}{\sigma - 1} \Leftrightarrow P(\mathbf{p}) = \text{constant} \times A(\mathbf{p})$$

- ▶ hence: for CES, the common aggregator and the ideal price index coincide!
- ▶ reason: for all goods, consumer surplus to sales ratio is constant, exogenous and equal to the sales to variable costs = variable profit ratio  $1/(\sigma - 1)$
- ▶ under general HSA: consumer surplus is variable, endogenous and may be higher or lower than the private surplus (i.e., the markup)

# Markups under HSA

- ▶ under HSA, demand elasticity is variable, even if producer  $\omega$  takes aggregates, i.e., the common aggregator  $A$  as given

$$\sigma_{\omega} = \sigma_{\omega} \left( \frac{p}{A} \right) = 1 - \frac{\partial \ln s_{\omega}(p/A)}{\partial \ln(p/A)}$$

- ▶ by choosing  $s_{\omega}(\cdot)$ , can match any shape of (downward-sloping) residual demand curve - hence, can match a lot (!) of patterns for the markup  $\mu_{\omega}$

$$\mu_{\omega} = \mu_{\omega} \left( \frac{p}{A} \right) = \frac{\sigma_{\omega} \left( \frac{p}{A} \right)}{\sigma_{\omega} \left( \frac{p}{A} \right) - 1}$$

- ▶ markups vary by relative price and may vary by  $\omega$  (conditional on relative price, e.g., quality may matter differentially from productivity)
- ▶ markups may be decreasing in relative price (as in Atkeson-Burstein), increasing in relative price, or display a non-linear relationship with price



# HSA: Functional form examples

- ▶ perturbed CES with monotonicity:  $\sigma(z) = \sigma + a(\sigma - 1)g(z)$ ,
  - here:  $g' > 0, g(0) = 0, g(\infty) = 1$ , and  $\sigma, a$  are constants
  - gives markup - sales relationship as Atkeson-Burstein if  $a < 0$  perturbed CES:  $\sigma(z) = \sigma + a(\sigma - 1)g(z)$ , where  $g' > 0, g(0) = 0, g(\infty) = 1$
- ▶ perturbed CES w/o monotonicity:  $\sigma(z) = 1 + (\sigma - 1) \frac{\delta z g'(z)}{1 + \delta(\sigma - 1)g(z)}$ 
  - here,  $g(0) = 0, g(\infty) = 0$
  - $\delta$  can be positive or negative: allows non-linear markup-sales patterns
- ▶ generalized translog:  $\sigma(z) = 1 + \eta / [\ln(\bar{z}/z)]$  for  $z < \bar{z}$ 
  - introduced to the trade literature by Feenstra 2003
  - famous for the choke price

# Pass-throughs under HSA

- ▶ let  $p_\omega = \mu_\omega \times mc_\omega$
- ▶ price-cost pass-through  $\rho_\omega$  depends (intuitively) on the elasticity of the markup function

$$\rho_\omega\left(\frac{p}{A}\right) \equiv \frac{\partial \log p_\omega}{\partial \log mc_\omega} = \frac{1}{1 - \frac{\frac{p}{P} \mu'_\omega\left(\frac{p}{P}\right)}{\mu_\omega\left(\frac{p}{P}\right)}}$$

- ▶  $\rho_\omega = 1$  whenever a firm  $\omega$  has a constant markup
- ▶ else,  $\rho_\omega \in [0, 1]$  can be guaranteed if  $\mu'_\omega < 0$ , i.e., if each firm's markup share is decreasing in its sales share

# Micro-foundation: The role of IIA

- ▶ similar to the CES demand system, the HSA demand system can be derived from a logit discrete choice model (Trottner 2023)
- ▶ however, in contrast to standard logit, choices will violate independence of irrelevant alternatives, i.e.: the differentiation between goods will depend on the overall competitiveness of the market
- ▶ IIA + infinitely many firms: no complementarities in pricing behavior
- ▶ IIA + finitely many firms: direct strategic complementarities between each pair of firms
- ▶ HSA + infinitely many firms: indirect strategic complementarities between any firm and all other firms

# HSA in Action: Melitz revisited

- ▶ consider a closed-economy, with mass of  $L$  consumers
- ▶ normalize  $w = 1$
- ▶ symmetric HSA, so total demand for each firm is  $Ls(\frac{p_\omega}{A})$ , CES case is

$$s^{CES}(\frac{p_\omega}{A}) = \left(\frac{p_\omega}{A}\right)^{1-\sigma}$$

- ▶ firms with productivity  $\varphi$  behave symmetrically, index firms by  $\varphi$
- ▶ technology: A firm with productivity  $\varphi$  can produce  $q$  using  $l$  units of labor according to

$$l(\varphi) = f_d + \frac{q(\omega)}{\varphi(\omega)}$$

- ▶ entry cost  $f_d$ , overhead cost  $f_o$

# HSA Melitz: pricing and profits

- ▶ HSA price

$$p_{\varphi} = \mu_{\omega} \left( \frac{p_{\omega}}{A} \right) / \varphi$$

- ▶ CES price:

$$p_{\varphi} = \mu / \varphi$$

- ▶ HSA operating profits under HSA

$$\pi_{\varphi} = L \left( 1 - \frac{1}{\mu_{\varphi}} \right) s_{\varphi} - f_o$$

- ▶ CES operating profits

$$\pi_{\varphi} = L \left( 1 - \frac{1}{\mu} \right) s_{\varphi}^{CES} - f_o$$

# Zero Profit Condition

- ▶ exit cutoff  $\varphi^*$  under HSA

$$L\left(1 - \frac{1}{\mu_{\varphi^*}}\right) s_{\varphi^*} = f_o$$

- ▶ exit cutoff under CES

$$L\left(1 - \frac{1}{\mu}\right) s_{\varphi^*}^{CES} = f_o$$

- ▶ difference: curvature of the markup function matters

# Free Entry

- ▶ Free entry condition under HSA

$$\int_{\varphi^*}^{\infty} \left[ L \left( 1 - \frac{1}{\mu_{\varphi}} \right) s_{\varphi} - f_o \right] dG(\varphi) = f_e$$

- ▶ exit cutoff under CES

$$\int_{\varphi_{CES}^*}^{\infty} \left[ L \left( 1 - \frac{1}{\mu} \right) s_{\varphi}^{CES} - f_o \right] dG(\varphi) = f_e$$

# Equilibrium: $M, A, \varphi^*$ s.t.

- ▶ goods market clear, exit and entry are optimal

(market clearing) 
$$M \int_{\varphi^*}^{\infty} s\left(\frac{p_{\omega}}{A(p)}\right) dG(\omega) = 1$$

(ZPC) 
$$L\left(1 - \frac{1}{\mu_{\varphi^*}}\right) s_{\varphi^*} = f_o$$

(Free entry) 
$$\int_{\varphi^*}^{\infty} \left[ L\left(1 - \frac{1}{\mu_{\varphi}}\right) s_{\varphi} - f_o \right] dG(\varphi) = f_e$$

where  $\mu_{\varphi} = \frac{\sigma_{\varphi}\left(\frac{p}{A}\right)}{\sigma\left(\frac{p}{A}\right) - 1}$ ,  $s_{\varphi} = s\left(\frac{p_{\varphi}}{A}\right)$ ,  $p_{\varphi} = \mu_{\varphi} / \varphi$

- ▶ guess  $A$ , solve for prices, markups and sales shares
- ▶ then solve for  $\varphi^*$  using ZPC and  $M$  using market clearing
- ▶ given all the above, solve for  $A$  using free entry, compare, update...



# Equilibrium: Efficiency

(market clearing) 
$$M \int_{\varphi^*}^{\infty} s\left(\frac{p_{\omega}}{A(p)}\right) dG(\omega) = 1$$

(ZPC) 
$$L\left(1 - \frac{1}{\mu_{\varphi^*}}\right) s_{\varphi^*} = f_o$$

(Free entry) 
$$\int_{\varphi^*}^{\infty} \left[ L\left(1 - \frac{1}{\mu_{\varphi}}\right) s_{\varphi} - f_o \right] dG(\varphi) = f_e$$

where  $\mu_{\varphi} = \frac{\sigma_{\varphi}\left(\frac{p}{A}\right)}{\sigma\left(\frac{p}{A}\right) - 1}$ ,  $s_{\varphi} = s\left(\frac{p_{\varphi}}{A}\right)$ ,  $p_{\varphi} = \mu_{\varphi} / \varphi$

- ▶ note that  $P$  does not appear in these equilibrium conditions
- ▶ intuitively: market mechanism incentivizes competition,  $A$ , through private profits,  $\mu_{\varphi}$ , but not welfare,  $P$ , which depends on “social profits”
- ▶  $A$  and  $P$  coincide only in the CES case
- ▶ hence: the equilibrium will be efficient if and only if markups are constant

# Local margins of inefficiency

- ▶ cross-sectional distortion:  $\varphi$  too small compared to  $\varphi'$  iff

$$\mu_{\varphi} > \mu_{\varphi'}$$

- ▶ entry distortion:  $M$  is excessive (else, insufficient) iff

$$\bar{\mu} = \left( \int_{\varphi^*}^{\infty} s_{\varphi} (1/\mu_{\varphi})^{-1} dG(\varphi) \right)^{-1} > \left( \int_{\varphi^*}^{\infty} s_{\varphi} \delta_{\varphi} dG(\varphi) \right) = \bar{\delta}$$

- ▶ selection distortion:  $\varphi^*$  is too high (else, too low) iff

$$\bar{\delta} > \delta_{\varphi^*}$$

- ▶ see also Baqaee Fahri 2023, Edmond Midrigan Xu 2023
- ▶ “sufficient statistics”: also when underlying heterogeneity goes beyond productivity

# Global Welfare Change: Ex-Post statistics

$$d \ln \frac{1}{P} = (\bar{\delta} - 1)d \ln M + (\delta_{\varphi^*} - \bar{\delta}) \frac{g(\varphi)}{1 - G(\varphi)} d\varphi^* + \mathbb{E}_s \left[ d \ln \mu_{\varphi} \right]$$

- ▶ three margins of welfare gains

1.  $d \ln M$ : entry

2.  $d\varphi^*$ : selection

3.  $d \ln \mu_{\varphi}$ : markups

- ▶ CES-baseline: all welfare effects from entry  $d \ln \frac{1}{P} = (\mu - 1)d \ln M$

- markups exogenous  $\mu_{\varphi} = \mu$ ,

- all varieties provide the same surplus  $\delta_{\omega} = \delta_{\omega'} = \mu$

- ▶ variable, heterogeneous markups

- all margins active

# Gains from Market Size in Krugman

- ▶ Krugman: homogeneous firms, no selection + cross-sectional distortions

market clearing:  $M s\left(\frac{p}{A}\right) = 1$ , (FE)  $L \left[ 1 - \frac{1}{\mu(p/A)} \right] s\left(\frac{p}{A}\right) = f_e$

$$-\ln P/A = M \int_{p/A}^{\infty} \frac{s(\xi)}{\xi} d\xi = M(\delta - 1)$$

$$\mu(p/A) = \frac{\sigma\left(\frac{p}{A}\right)}{\sigma\left(\frac{p}{A}\right) - 1}, p = \mu$$

- ▶ can show that welfare effect of market size  $d \ln L$  equals

$$d \ln \frac{1}{P} = (\delta - 1) d \ln L + \left( 1 - \frac{\delta}{\mu} \right) \sigma(1 - \rho) d \ln L$$

- ▶  $\Delta$  technical efficiency “love-for-variety” +  $\Delta$  allocative efficiency

# Gains from Market Size: Krugman

$$-\ln P/A = M \int_{p/A}^{\infty} \frac{s(\xi)}{\xi} d\xi = M(\delta - 1)$$

- ▶ welfare effect of market size  $d \ln L$  equals

$$d \ln \frac{1}{P} = (\delta - 1)d \ln L + \left(1 - \frac{\delta}{\mu}\right) \sigma(1 - \rho)d \ln L$$

- ▶  $\Delta$  technical efficiency “love-for-variety” +  $\Delta$  allocative efficiency
- ▶ when demand is CES, then  $\delta = \mu$ ,  $\rho = 1$ , so only technical gains
  - gains reflect pure increase in variety  $M$ , which consumers “love”
- ▶ else, increase in  $M$  can “on the margin” improve allocative efficiency if:
  - competition lowers the markup  $1 - \rho > 0$  and entry was excessive  $\mu > \delta$
  - competition raises markup  $1 - \rho < 0$  and entry was insufficient  $\mu < \delta$

# Gains from Market Size: Melitz

- ▶ get two additional allocative effects
  - cross-sectional distortions: conditional on markups, high-markup firms are more shielded from competition, so entry (i.e., competition) reallocates towards them, which improves efficiency (Baqae Fahri 23: Darwinian effect)

$$d \ln c_{\varphi} = -\sigma_{\varphi} d \ln \frac{\mu_{\varphi}}{A} - d \ln A$$

- selection distortion: depends on whether selection was initially too high/low and whether competition induces more or less entry
- ▶ key lesson: returns-to-scale are endogenous when markups are variable
- ▶ (local) returns-to-scale are the single-most important statistic in growth (i.e., Romer) and spatial (i.e., place-based policies)

# Another homothetic non-CES demand system

- ▶ Kimball (HDIA in Matsuyama's classification): defined implicitly

$$\int_{\Omega} \phi_{\omega}\left(\frac{c_{\omega}}{C(\omega)}\right) = 1$$

- ▶ demand system requires two aggregators,  $P(\mathbf{p})$  and  $B(\mathbf{p})$

$$\frac{c_i p_i}{E} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_{\omega}} = \frac{p_{\omega}}{P(\mathbf{p})} (\phi'_{\omega})^{-1} \left( \frac{p_{\omega}}{B(\mathbf{p})} \right)$$

$$\int_{\omega} \phi_{\omega} \left( (\phi'_{\omega})^{-1} \left( \frac{p_{\omega}}{B(\mathbf{p})} \right) \right) d\omega = 1$$

$$P(\mathbf{p}) = \int p_{\omega} (\phi'_{\omega})^{-1} \left( \frac{p_{\omega}}{B(\mathbf{p})} \right) d\omega$$

- ▶  $B(\mathbf{p})$  is a competition index (similar role to  $A$  in HSA)
- ▶ similarly flexible to HSA, CES is special case if  $\phi_{\omega}(x) = x^{(\sigma-1)/\sigma}$
- ▶ two aggregators can be cumbersome, but its not too bad, Kimball has seen a LOT of usage in short-run macro

# Klenow-Willis functional form

- ▶ Klenow-Willis 2016 proposed the by far most popular functional form for Kimball's 1995 aggregator

$$\phi(x) = (\bar{\sigma} - 1) \exp\left(\frac{1}{\epsilon}\right) \epsilon^{\frac{\bar{\sigma}}{\epsilon} - 1} \left[ \Gamma\left(\frac{\bar{\sigma}}{\epsilon}, 0\right) - \Gamma\left(\frac{\bar{\sigma}}{\epsilon}, x^{\frac{\bar{\sigma}}{\epsilon}} / \epsilon\right) \right]$$

- ▶  $\Gamma(\cdot, \cdot)$  is the incomplete Gamma function,  $\bar{\sigma} > 1, \epsilon > 0$

- ▶ implied elasticity:  $-\frac{\partial \ln c\left(\frac{p_{\omega}}{B}\right)}{\partial \ln\left(\frac{p_{\omega}}{B}\right)} = \sigma\left(\frac{p_{\omega}}{P}\right) = \frac{\bar{\sigma}}{1 - \epsilon \log\left(\frac{\sigma - 1}{\sigma} \frac{p_{\omega}}{B}\right)}$

- ▶ super-elasticity:  $-\frac{\partial \ln \sigma\left(\frac{p_{\omega}}{B}\right)}{\partial \ln\left(\frac{p_{\omega}}{B}\right)} = \frac{\epsilon}{1 - \epsilon \log\left(\frac{\sigma - 1}{\sigma} \frac{p_{\omega}}{B}\right)}$

- ▶ relationship between share of variable cost in sales and markups:

$$\ln \frac{1}{\mu_{\omega}} + \ln\left(1 - \frac{1}{\mu_{\omega}}\right) = \text{constant} + \frac{\epsilon}{\sigma} \ln s_{\omega}$$



# Kimball in the Literature

- ▶ Edmund, Midrigan, Xu: How costly are Markups?, JPE, 23
- ▶ Baqaee, Farhi, Sangani: The supply-side effects of MP, JPE 23
- ▶ Santiago Franco, Output Market Power and Spatial Misallocation 24 JMP
- ▶ Werning, Wang: Dynamic Oligopoly, AER 22
- ▶ Edmund, Midrigan, Xu: Competition, Markups, and Gains from International Trade, AER 2015
- ▶ Gopinath, Itskhoki: Currency Choice and Exchange Rate Pass-through, AER 10
- ▶ ...

# Summary

- ▶ Non-CES MC is a highly tractable way to account for variable markups with free entry
- ▶ cost: does not speak to the highly concentrated nature of many markets as firms are still atomistic
- ▶ homothetic non-CES aggregators imply that the value of having goods available at a certain set of prices to consumers depends on market-wide price statistic(s)
- ▶ comparison: in search, reservation price is a function of the distribution of prices
- ▶ if price distributions can be characterized by a handful of statistics (i.e., mean, variance), could we perhaps map non-CES demand systems into those arising in search settings?