Ricardian Models

ECON245 - Winter 24

Overview

- David Ricardo (1817): Introduces the idea of comparative advantage
  - Setup: Two goods two countries, perfect competition + productivity diff.
  - Prediction: Countries export the good which is cheaper in autarky
  - Gains from trade, even if one country is strictly better at everything
- Limited explorations of this idea: Hard to study in settings with many goods and countries
- "The Ricardian model became something like a family heirloom, brought down from the attic to show a new generation of students, and then out back allowing them to pursue more fruitful lines of study and research" Eaton and Kortum, 2012
- 1977: Dornbusch, Fischer & Samuelson develop a tractable Ricardian model of trade with infinitely many goods

Dornbusch-Fischer-Samuelson

- "smooths" the Ricardian model by introducing many goods, $z \in [0,1]$
- Cobb-Douglas preferences: $U_i = \int_0^1 \log y_i(\omega) dz$,
- $A_H(z), A_F(z)$: productivity at which countries H and F produce z
- comparative advantage: $A(z) \equiv A_H(z)/A_F(z)$, assumed to be strictly decreasing and convex - H's comparative advantage is decreasing in z
- market structure: perfect competition + symmetric trade costs τ, competitive wages with wages w_i
- price of good z if made in i and consumed in j is $p_{ij}(z) = \tau w_i / A_i(z)$

Trade patterns + equilibrium

- each good is sourced from the cheapest supplier: H will import z iff

$$w_H/A_H(z) \geq \tau w_F/A_F(z) \Leftrightarrow w_H/[w_F\tau] \geq A(z)$$

- hence: H will import goods $z \in [z_H^*, 1]$, where $A(z_H^*) = w_H/[w_F\tau]$
- F will import goods $z \in [0, z_F^*]$, where $A(z_F^*) = \tau w_H/w_F$
- market clearing pins down wages:

$$w_H L_H = w_H L_H z_H^* + w_F L_F z_F^*$$

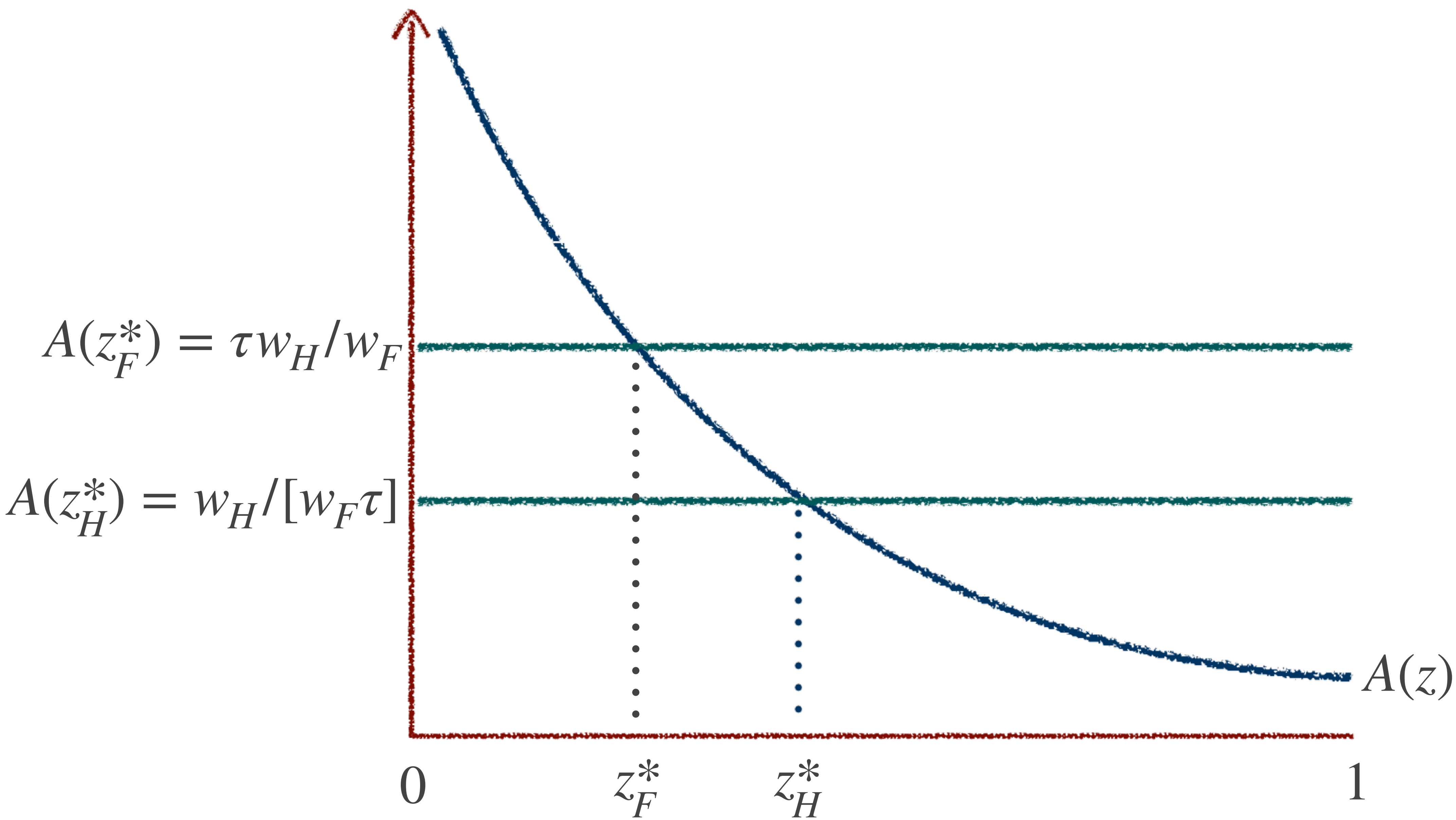
$$w_F L_F = w_F L_F \left[1 - z_F^*\right] + w_H L_H \left[1 - z_H^*\right]$$

- can show: welfare in H is higher when z_H^* is lower

$$w_H/P_H = A(z_H^*)\left[\int_O^{z_H^*} A(z_H^*)/A_H(z)dz + \int_{z_H^*}^1 1/A_F(z)dz]\right]^{-1}$$

Trade patterns: graphically

- comparative advantage: relative productivities determine trade
- goods in $[z_F^*, z_H^*]$ are not traded



From DFS to Eaton-Kortum 2002

- DFS: Continuous goods space + 2 countries
- Model still does not extend to many-country settings
- Issue: Discrete choice
  - With N countries, there are N2 possible combinations of production patterns for each good
  - Similar combinatorial issues in any model of discrete choice
- Solution: Probabilistic formulation, i.e., random discrete choice
  - Intuition: For each ω, will import from j with some probability
  - continuum of goods: Computing aggregate flows, by the Law of Large Numbers, boils down to calculating moments from a distribution
- Challenge: Characterizing this (endogenous) distribution of choic

Eaton-Kortum 2002

- A probabilistic formulation of the Dornbusch-Fisher-Samuelson model
    - Comparative advantage differences promote trade
    - Geographic barriers diminish trade
- The probabilistic formulation itself is a hugely influential technical contribution
- Underlying technique applies in many discrete-choice setting: Job choice, location choice, commuting choice, school choice…

Model Set-up

- countries are indexed by $i \in 1,...,N$
- continuum of goods $\omega \in [0,1]$
- labor only factor of production, L_i workers in each country
- CES preferences over varieties with elasticity σ
- constant returns to scale production with $z_i(\omega)$ productivity of variety ω in country i
- Iceberg trade costs τ_{ij} from country i to j, where $\forall i, \tau_{ii} = 1$.

Preferences

- consumers have CES preferences over the set of varieties $\omega \in [0,1]$
- Each variety is ω is homogeneous across countries
- perfect competition, so prices equal marginal cost

$$p_{ij}(\omega) = \frac{w_i}{z_i(\omega)}\tau_{ij}$$

- consumers in each country shop for the cheapest source country to buy each variety ω, so the price paid for ω in destination j equals

$$p_j(\omega) = \min_{i=1,\ldots,N} \{p_{ij}(\omega)\}$$

Technology

- country $i's$ efficiency in producing variety ω is the realization of a random variable Z_i drawn from $F_i(z) = Pr(Z_i \leq z)$
  - by LLN, $F_i(z)$ is the fraction of varieties for which i has efficiency below z
- Eaton-Kortum choose F to be the Frechet distribution

$$F_i(z) = exp(T_i z^{-\theta}), T_i > 0$$

- T_i captures absolute comparative advantage of country i
- θ is an (inverse) measure of the degree of comparative advantage
  - intuition: lower θ_i indicates more dispersion.
  - more dispersion spells more specialization
  - i.e., if $\theta \to \infty$, then $\ln z_i(\omega) \to T_i$ almost surely

Key Property of Extreme Value Distributions

- distributions in the class of extreme value distributions are “max and min stable”
  - E.g.: Frechet, Gumbel, Weibull
- in other words: The minimum or maximum of a list of i.i.d. Frechet variables follows a freshet distribution

$X_{min} = \min\{x_1, x_2, \ldots, x_n\}$ and $x_i \sim$ Frechet then $X_{min} \sim$ Frechet

- extreme Value Theorem: Only extreme value distributions are “max and min stable”
- this property is very useful to economists

Prices

- origin country i presents a destination j with a distribution of prices $G_{ij}(p) = Pr[p_{ij} \leq p] = 1 - F_i(w_i \tau_{ij}/p)$:

$$G_{ij}(p) = 1 - \exp\left(-\left[T_i(w_i \tau_i j)^{-\sigma}\right] p^{-\sigma}\right)$$

- the distribution of the minimum of prices (i.e. the actual price paid by consumers) in destination j for any variety is

$$G_j(p) = Pr(p_j \leq p) = Pr[\min_i p_{ij}(\omega) < p)] = 1 - \prod_{i=1}^{N} [1 - G_{ij}(p)]$$

- substituting $G_{ij}(p)$ yields:

$$G_j(p) = 1 - \exp(-\Theta_j p^{\theta}) \text{ where } \Theta_j \equiv \sum_i T_i(w_i \tau_{ij})^{-\theta}$$

Corollaries of the Frechet assumption

- the probability that country i provides a given good at the lowest price:

$$Pr[p_{ij} \leq \min_{k \neq i} p_{kj}] \equiv \lambda_{ij} = \frac{T_i(w_i \tau_{ij})^{-\theta}}{\Theta_j} = \frac{T_i(w_i \tau_{ij})^{-\theta}}{\sum_i T_i(w_i \tau_{ij})^{-\theta}}$$

  - LLN: this is also the fraction of goods j purchases from i

- the price of a good that country n actually buys from any country i also has the distribution $G_i(p)$

$$P\left[p_{ij}(\omega) < p \,|\, i = \arg\min_n p_{nj}(\omega)\right] = G_j(p)$$

  - so the distribution of prices is the same for any source country
  - what differs, is the cardinality of the set of goods sourced

- hence, λ_{ij} also equals the share of j's expenditures on goods from i

What is the relevant trade elasticity?

- let $\Omega_{ij} = \{\omega : i = \arg\min_n p_{nj}(\omega)\}$ denote bundle of goods that j ends up buying from i
- the consumer price index in each country i can be written

$$P_j = \left(\sum_i P_{ij}^{1-\sigma} \right)^{1/(1-\sigma)}, \text{ where } P_{ij} \equiv \left(\int_{\Omega_{ij}} p_{ij}(\omega)^{1-\sigma} d\omega \right)^{1/1-\sigma}$$

  - looks like like Armington - except Ω_{ij} is endogenous

- LLN: $P_{ij} = \lambda_{ij}^{1/(1-\sigma)} \times \mathbb{E}\left[p_{ij}(\omega)^{1-\sigma} \,|\, i = \arg\min_n p_{nj}(\omega) \right]^{1/(1-\sigma)}$
- optimally sourced goods have the same distribution, independently of their source country $\Rightarrow$ $P_{ij} = \lambda_{ij}^{1/(1-\sigma)} \mathbb{E}_{G_j}\left[p^{1-\sigma} \right]^{1/(1-\sigma)}$

What is the relevant trade elasticity?

- using the previous steps, it is easy to show:

$$P_j = \left(\sum_i \lambda_{ij} \mathbb{E}_{G_j} [p]^{1-\sigma} \right)^{1/(1-\sigma)} = \mathbb{E}_{G_j} \left[p^{1-\sigma} \right]^{1/(1-\sigma)}$$

- result: if $x \sim F(x) = 1 - e^{-Ax^\theta}$, then $\forall \rho > -\theta$, $\mathbb{E}[x^\rho] = A^{-\rho/\theta} \Gamma \left(\frac{\theta + \rho}{\theta} \right)$
  - $\Gamma(\cdot)$ is the Gamma function, $\theta > \rho$ ensures the mean is defined
- hence: $P_j = \mathbb{E}_{G_j} \left[p^{1-\sigma} \right]^{1/(1-\sigma)} = constant \times \mathbb{E}_{G_j}[p_j]$
- lesson: σ is irrelevant for how P_j + expenditure allocations respond to price changes
  - σ captures flexibility to adjust expenditures on the intensive margin
  - in Armington, trade flows only adjust on the intensive margin
  - in EK, extensive margin adjustment always ensures that all goods purchased have the same price (in a LLN sense), independently of their origin
  - hence, value of σ is irrelevant for expenditure allocations and welfare
  - θ captures flexibility of extensive margin adjustment, and hence the trade elasticity

Corollaries of the Frechet assumption

- the price index in country j equals

$$P_j = \bar{\Gamma} \times \Theta_j^{-1/\Theta} = \bar{\Gamma} \times \left(\sum_i T_i (w_i \tau_{ij})^{-\theta} \right)^{-1/\theta}$$

where $\bar{\Gamma} \equiv \Gamma(\frac{\theta + 1 - \sigma}{\theta})^{1/(1-\sigma)}$

- $\Theta_j^{-1/\theta}$ captures access to cheap consumables
  - summarizes technology, input costs, and geographic barriers around the world
  - looks similar to the price index in Armington, but the welfare relevant elasticity is now θ

Aside: Footnote 15, DFS and EK

- $A(z)$ captured comparative advantage of the home economy in DFS
- one interpretation: $A(z)$ as the fraction of goods for which the ratio of home to foreign efficiency is at least A
- with two countries $N = 2$, one can show that Frechet-distributed productivities imply

$$A(x) = (T_1/T_2)^{1/\theta}\left[(1 - x)/x\right]^{1/\theta}$$

- this shows that for two countries, Eaton-Kortum is simply a special case of DFS
- however, for more than two countries, EK remains highly tractable, while DFS does not

Trade flows and Gravity

- Recall that the fraction of goods sourced from any origin is also the share of total spending devoted to that origin!
- Total volume of production in country i

$$Q_i = \sum_j X_{ij} = \sum_j \lambda_{ij} X_j = T_i w_i^{-\theta} \sum_j \frac{\tau_{ij}^{-\theta} X_j}{\Theta_j} = T_i w_i^{-\theta} \gamma \sum_j X_j \left(\frac{\tau_{ij}}{P_j}\right)^{-\theta}$$

$\equiv \Pi_i$

$$X_{ij} = \lambda_{ij} X_j = \frac{T_i w_i^{-\theta} \tau_{ij}^{-\theta} \gamma}{P_j^{-\theta}} X_j$$

- Solving for $T_i w_i^{-\theta}$ in the first, and substituting into the second equation yields the gravity equation:

$$X_{ij} = \tau_{ij}^{-\theta} \left(\frac{Q_i}{\Pi_i^{-\theta}}\right) \left(\frac{X_j}{P_j^{-\theta}}\right)$$

Gravity

$$X_{ij} = \tau_{ij}^{-\theta} \left(\frac{Q_i}{\Pi_i^{-\theta}} \right) \left(\frac{X_j}{P_j^{-\theta}} \right)$$

- interpretation: Π_i captures market access by producers, P_j captures market access by consumers

$$\ln X_{ij} = -\theta \ln \tau_{ij} + \ln Q_i + \ln X_j + \theta(\ln \Pi_i + P_j)$$

- the last term captures multilateral resistance
  - trade depends on bilateral resistance, but also on the importers access to consumables and the exporters access to consumers
- same reduced-form equation as in Armington
  - key difference is the interpretation of the trade elasticity: θ captures technology, in Armington $\sigma - 1$ captured preferences

Equilibrium

- All endogenous objects can be expressed as a function of $\{w_i\}_i$
- Goods market provides a systems of N equations in N variables

$$w_i L_i = \sum_j \lambda_{ij} w_j L_j$$

- Simple iterative procedure can solve efficiently for w

1. Guess wages
2. Compute trade shares λ_{ij} given the guess
3. Compute new wages implied by the equation above.
4. Compare to initial guess, update and repeat.

Welfare

- rearranging λ_{ii} we obtain:

$$\lambda_{ii} = \Gamma \frac{T_i w_i^{-\theta}}{P_i^{-\theta}} \Rightarrow \frac{w_i}{P_i} = \Gamma (\frac{T_i}{\lambda_{ii}})^{\frac{1}{\theta}}$$

- gains from trade show up in own-trade share
  - reflects revealed preference: “How much am I borrowing abroad’s technology?”
  - gains greater the more dispersed technology draws are
- λ_{ii} and θ are sufficient to calculate welfare changes in response to changes in fundamentals in any other country and any trade cost.
  - E.g., going from baseline (1990) to autarky $\pi_{nn} = 1$ implies losses between -0.2% and -10% (smallest for Japan and US (-0.8%).

Welfare in EK vs Armington

- welfare in Eaton Kortum

$$\frac{w_i}{P_i} = \Gamma \left(\frac{T_i}{\lambda_{ii}} \right)^{\frac{1}{\theta}}$$

- welfare in Armington

$$\frac{w_i}{P_i} = \left(\frac{a_{ii} A_i^{\sigma-1}}{\lambda_{ii}} \right)^{1/(\sigma-1)}$$

- in either case, changes in home expenditure shares and the trade elasticity are sufficient for welfare analysis of trade shocks

Hat-Algebra

- issue for calibration: even the basic version of the model has $N \times N \times N + 2$ parameters
- Dekle et al 07: let $\hat{x} = x'/x$ denote the change in a variable x
- can rewrite equilibrium conditions in terms of changes in wages, given an initial allocation $\{\lambda_{ij}, w_i L_i\}_{ij}$ and changes in exogenous parameters (trade costs, technology, factor endowments)

$$\hat{\lambda}_{ij} = \lambda_{ij} \hat{T}_i (\hat{\tau}_{ij} \hat{w}_{ij})^{-\theta}$$

$$w_i L_i \hat{w}_i \hat{L}_i = \sum_j \lambda_{ij} w_j L_j \hat{\lambda}_{ij} \hat{w}_i \hat{L}_j$$

- counterfactuals can be computed using data (initial trade shares, incomes) and require only one structural elasticity (per sector)
  - solve for $\hat{w}_i$, given counterfactual changes in fundamentals