Production Networks & Aggregate Fluctuations

ECON 245 - Winter 24

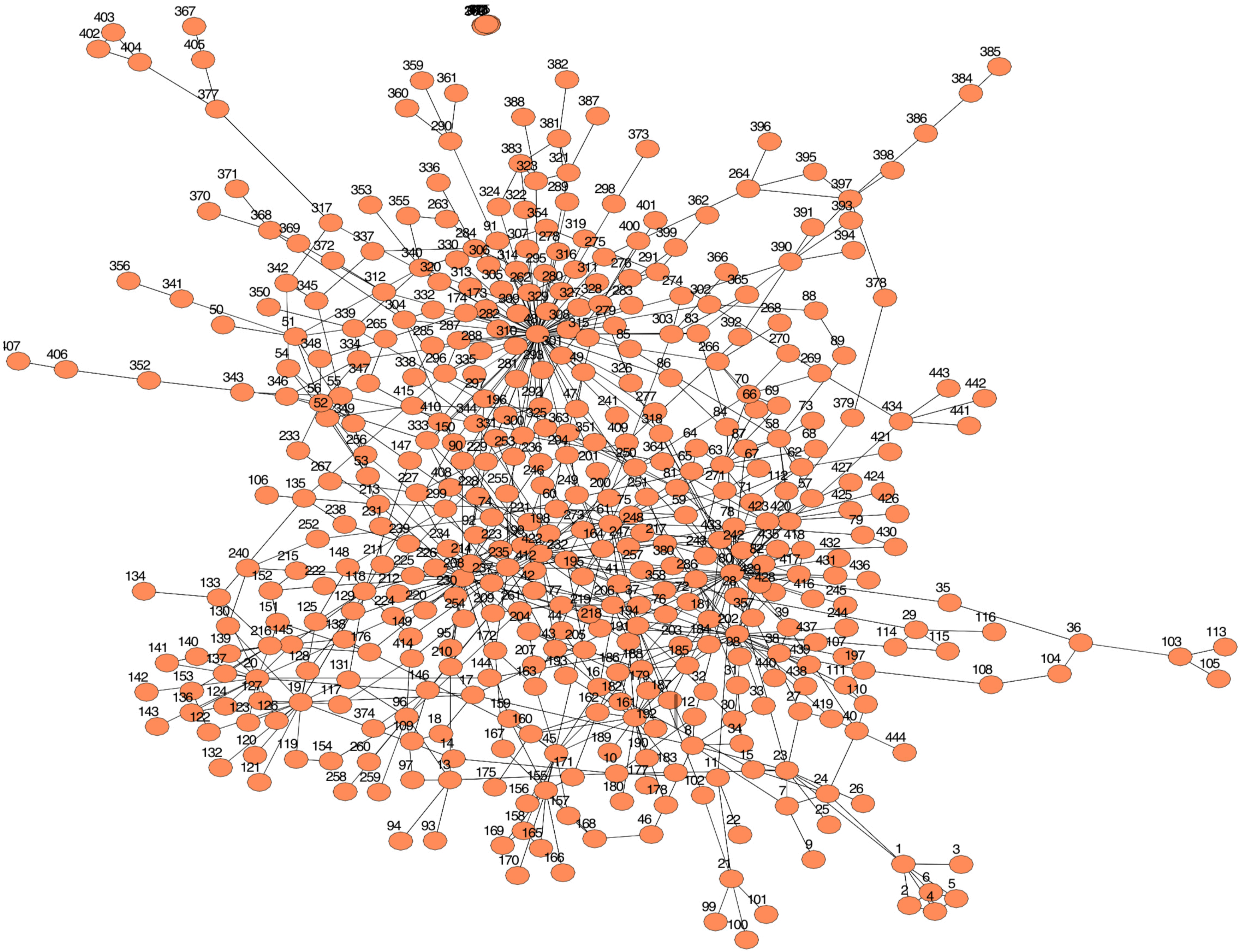
Outline

- **Q:** How do microeconomic shocks (to firms, sectors, regions) affect aggregate outcomes**?**
- **Plan:**
  - Theory: Hulten's Theorem and its applications/extensions
  - Empirical work

Production Networks

- Sectors (and firms) use each others' products as intermediate inputs
- This creates linkages (interconnections) across sectors and firms

Production Network: 1997 U.S. Economy



Micro Shocks and Macro Outcomes

- Given the complex network of interconnections between producers, how do shocks to individual producers affect aggregate
  - output?
  - productivity?
  - volatiltiy?
- Can micro shocks propagate to cause large aggregate movements?
  - A negative shock to one firm/sector affects input cost of all its costumers
  - This affects costumers of these firms/sectors, and so on...
- Now: canonical setup and Hulten's Theorem

Canonical Framework: Set-up

- Static economy with $1,...,n$ industries (could also be firms)
- Preferences of the representative household

$$u(c_1, \ldots, c_n) = \sum_{i=1}^{n} \beta_i \log(c_i/\beta_i)$$

where $\sum_i \beta_i = 1$

- Households supplies one unit of labor inelastically at wage w
- Perfect competition in output markets

Canonical Framework: Production

- Output of industry i

$$y_i = z_i \zeta_i l_i^{a_i} \prod_{j=1}^{n} x_{ij}^{a_{ij}}$$

  - l_i: labor input
  - x_{ij}: Inputs from sector j
  - z_i: Productivity
  - $A \equiv \{a_{ij}\}_{i,j}$: Input-Output matrix, cost shares in sales ("the network")

$$\forall \text{i}, \ \sum_j a_{ij} = 1 - a_i$$

  - ζ_i: Normalization constant

Canonical Framework: Equilibrium

- Profit-maximization by firms
- Utility-maximization by households
- Goods market clearing: For all i,

$$c_i + \sum_j x_{ji} = y_i$$

Canonical Framework: Optimal price

- Profit maximization implies the following price for sector i

$$\log(p_i/w) = \sum_{j=1}^{n} a_{ij} \log(p_j/w) - \log(z_i)$$

- Letting $\hat{p} = \{\log(p_i/w)\}_i$ and $\epsilon = \{\log z_i\}_i$:

$$\hat{p} = A\hat{p} - \epsilon \Rightarrow \hat{p} = -(I - A)^{-1}\epsilon$$

- For $i \neq j$, element ij of the Leontief inverse $(I - A)^{-1}$ is given by:

$$l_{ij} \equiv a_{ij} + \sum_{r=1}^{n} a_{ir}a_{rj} + \dots$$

measures the importance of industry j as direct and indirect supplier to industry i

$$\hat{p}_i = -\sum_{j} l_{ij}\epsilon_j$$

Canonical Framework: Domar weights

- Market clearing + Demands $p_j c_j = \beta_j w, p_j x_{ij} = a_{ij} p_i y_i$

$$p_j y_j = \beta_j w + \sum_i a_{ij} p_i y_i$$

- Domar weights: $\lambda_i = p_i y_i / w = p_i y_i / GDP$

$$\lambda_j \equiv \frac{p_j y_j}{w} = \beta_j + \sum_i a_{ij} \lambda_i \Leftrightarrow \lambda = (I - A')^{-1} \beta$$

- Domar weight is higher if a sector is an important supplier to sectors whose output is in high demand

$$\lambda_j = \sum_i l_{ij} \beta_i$$

Canonical Framework

Theorem *The log output of industry i is given by*

$$\log(y_i) = \sum_{j=1}^{n} l_{ij}\epsilon_j + \delta_i,$$

where $exp(\delta_i) = \sum_j l_{ji}\beta_j$ is a constant.

- Shocks in one industry propagate to other industries through the network
- This propagation through the network is captured by the elements of the Leontief inverse
- Productivity shocks propagate “downstream”

Canonical Framework

Theorem *The economy's log real value added is given by*

$$\log(GDP) = \sum_{i=1}^{n} \lambda_i \epsilon_i, \text{ where } \lambda_i = \frac{p_i y_i}{GDP} = \sum_{j=1}^{n} \beta_j l_{ji}$$

Proof:

Cost minimization: $\log w = \sum_j l_{ij} \epsilon_j + \log p_i$

Multiply by β_i and sum over all sectors:

$$\log w = \sum_{ij} \beta_i l_{ij} \epsilon_j + \sum_i \beta_i \log p_i$$

Normalize the consumer price index so that second term disappears.

Note that $\sum_{ij} \beta_i l_{ij} \epsilon_j = \sum_j \varepsilon_j \sum_i l_{ij} \beta_i$, then use $\lambda_i = \sum_j l_{ji} \beta_j$

Hulten's Theorem

Theorem *The economy's log real value added is given by*

$$\log(GDP) = \sum_{i=1}^{n} \lambda_i \epsilon_i, \textit{ where } \lambda_i = \frac{p_i y_i}{GDP} = \sum_{j=1}^{n} \beta_j l_{ji}.$$

- Output is a linear combination of industry-level productivities
- λ_i: Sufficient statistic for how shocks to one industry impact aggregate output (summarizes all paths in the I-O network and demand!)
  - Sales are easily observable - and the network is irrelevant?
- Corollary: Hulten's famous theorem

$$\frac{d\log(GDP)}{d\log z_i} = \lambda_i$$

- This result is much more general than it seems (as we will see)

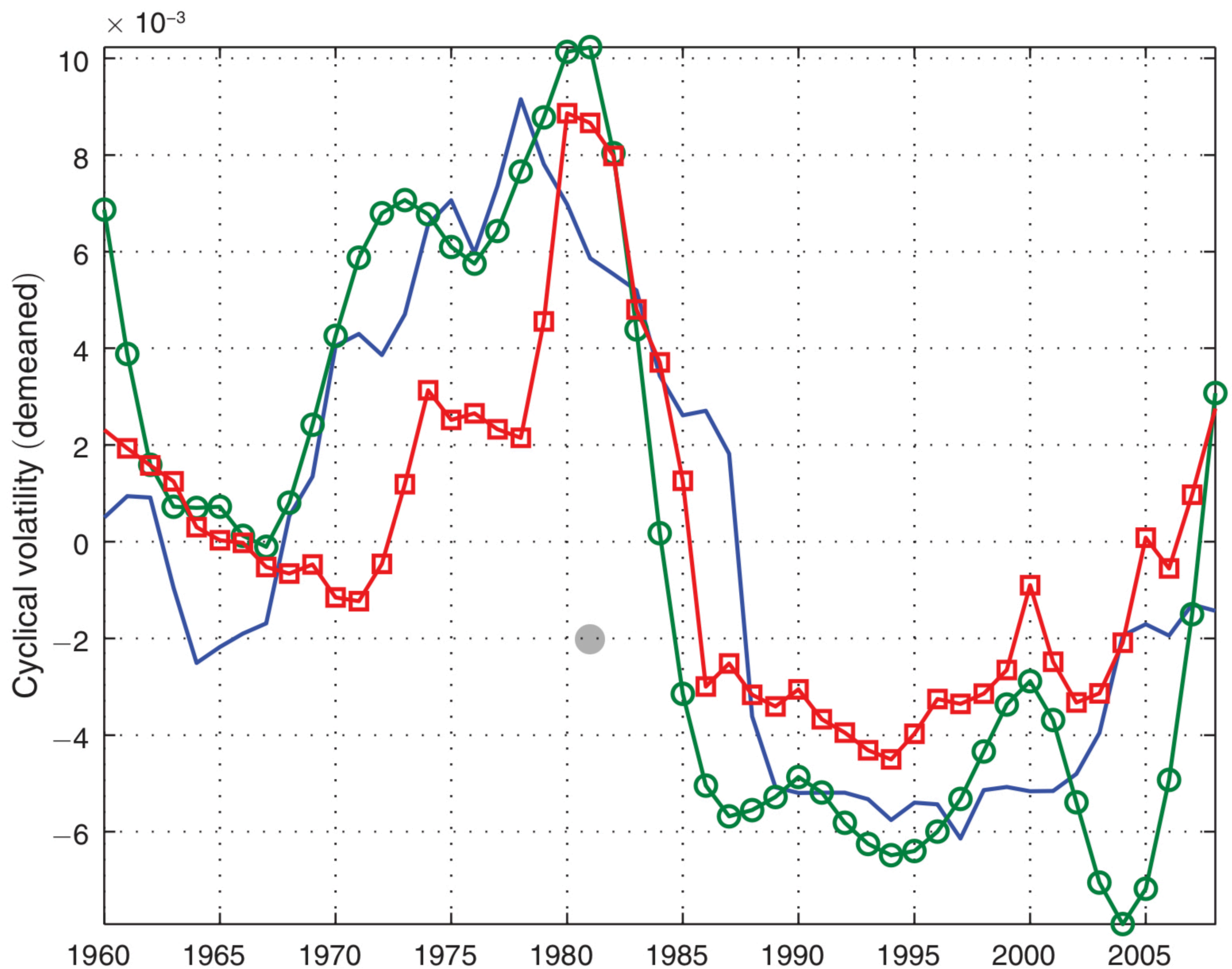
Applications: Carvalho Gabaix (AER 2013)

- Suppose that shocks ε_i are distributed i.i.d. and with variance σ_i
- Then aggregate volatility of GDP is given by:

$$\sigma = \sqrt{\sum_{i}^{n} \lambda_i^2 \sigma_i^2}$$

- CG show that changes in λ_i can explain decline and subsequent increase in U.S. GDP volatility
- They call σ, scaled by 4.5, "fundamental volatility"

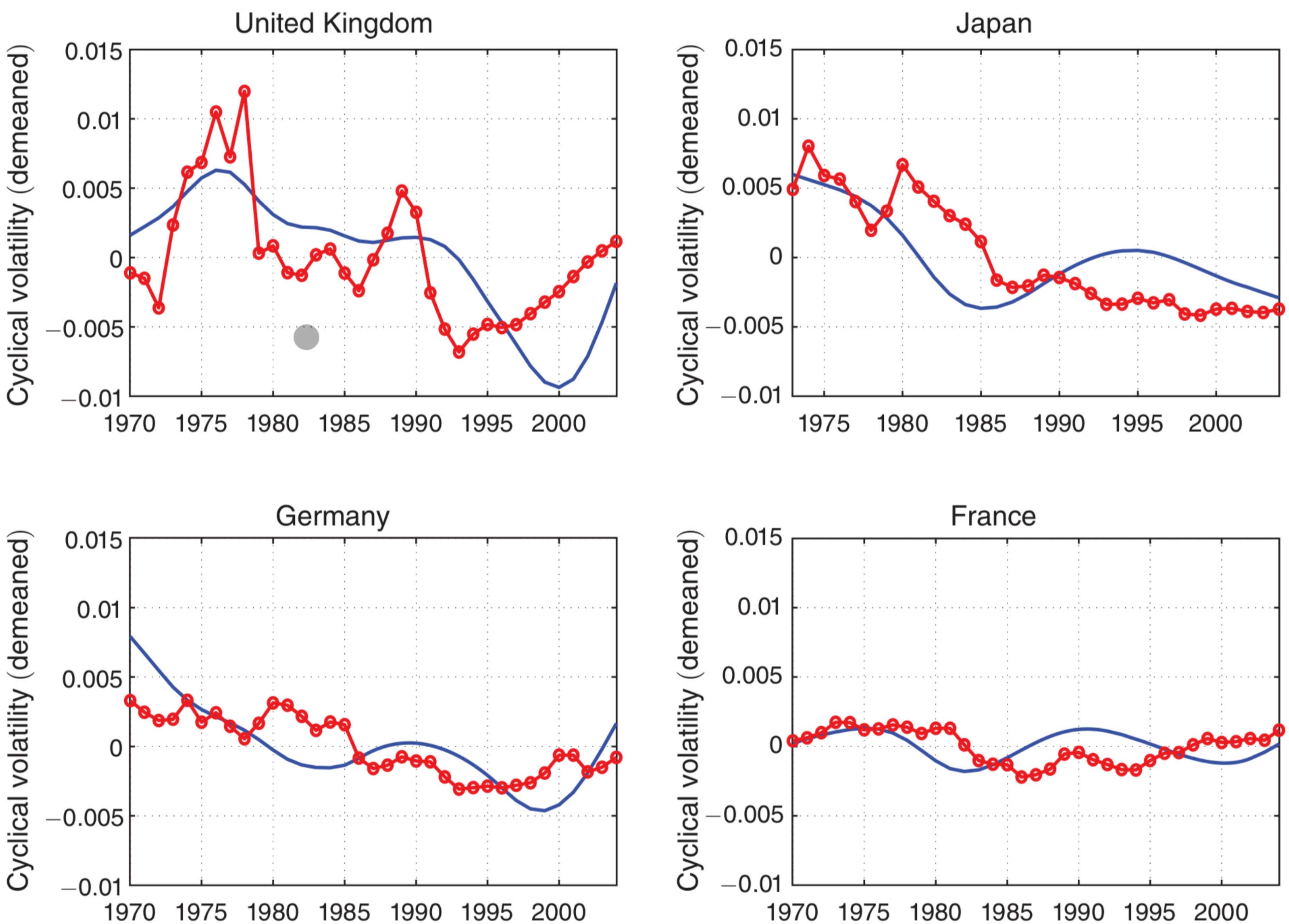
Carvalho Gabaix: U.S. evidence



Fundamental volatility. Rolling window HP-filter implied volatility of actual GDP

Carvalho Gabaix: International evidence

- Fundamental volatility vs actual GDP volatility



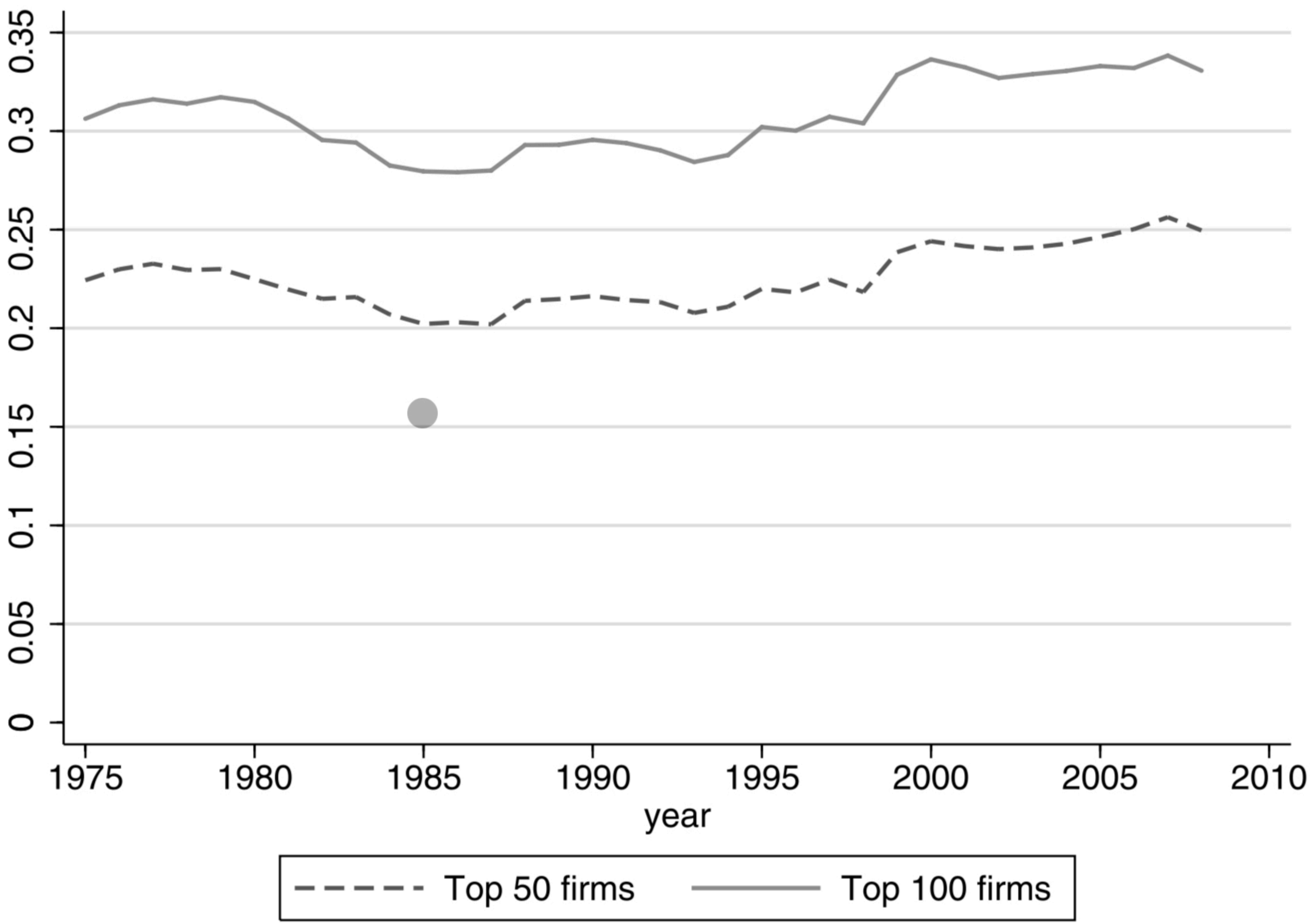
Applications: Irrelevance of Micro-shocks

- Assume firms instead of sectors, abstract from linkages
- Suppose that:
  - All sectors/firms are initially of identical size $S_i/S = 1/n$
  - all sector/firm shocks are independent with s.d. σ
- Then from Hulten's Theorem, we get:

$$\sigma_{GDP} = \sqrt{\sum_i \lambda_i^2 \sigma^2} = \frac{\sigma}{\sqrt{n}},$$

- Gabaix notes that $\sigma = 12\,\%$, $n = 10^6$, so $\sigma_{GDP} = 0.012\,\%$
- So idiosyncratic shocks are irrelevant?

Domar weights of the largest firms



Gabaix (2011): Fat tails break irrelevance

Proposition: *Consider an economy with n firms whose sizes are drawn from a power law distribution*

$$P(S > x) = ax^{-\zeta}$$

with exponent $\zeta \geq 1$. Suppose that all firms have the same volatility σ. Then as n goes to infinity, σ_{GDP} follows

$$\sigma_{GDP} \sim \frac{c}{\ln n}\sigma, \text{ for } \zeta = 1$$
$$\sigma_{GDP} \sim \frac{c}{n^{1/2}}\sigma, \text{ for } \zeta \geq 2$$

Comments

- One can think of Gabaix's result as providing a simple theory of the distribution of firm size, e.g. a theory of λ_i
- But alternatively, one could simply look at λ_i in the data and apply Hulten's theorem assuming firms are subject to i.i.d. shocks

$$\sigma_{GDP} = \sigma\sqrt{\sum_f \lambda_f^2}$$

- Implies $\sigma_{GDP} = 0.63\,\% >> 0.012\,\%$

Application: Acemoglu et al (ECMA 2012)

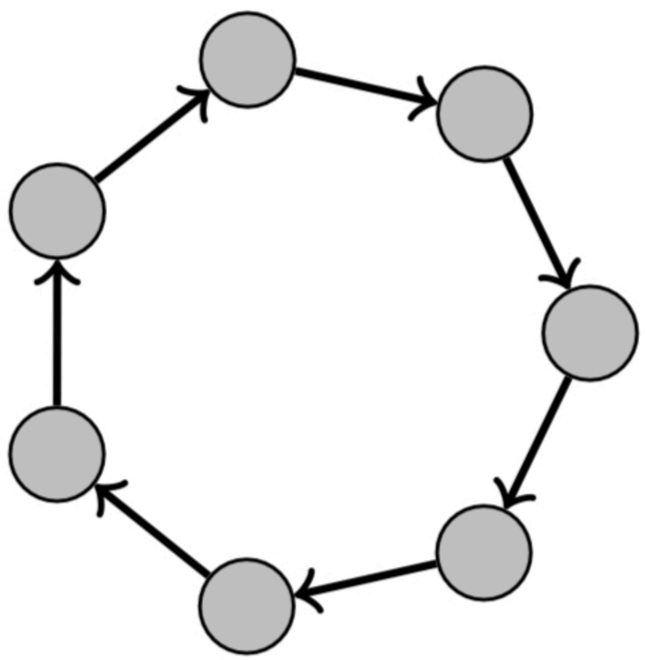
- uses the canonical framework, assuming equal value added shares $a_i = a$ across industries *and* Cobb-Douglas preferences with equal weights $\beta_i = 1/n, \forall i$.
- Then our previous results imply that Domar weights are given by:

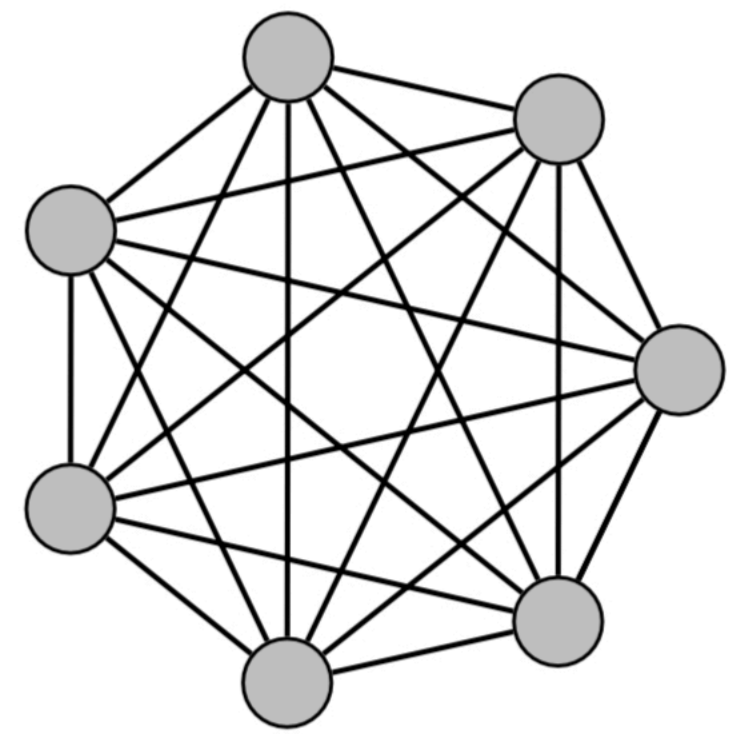
$$\lambda_i = \frac{p_i y_i}{GDP} = \sum_j \frac{1}{n} l_{ji}$$

- In network theory, this is called the Bonacich centrality index
- Can then use results from network theory to characterize how network structures affect the propagation of micro shocks
- Idea: Network structure maps into distribution of λ_i

Regular network: $d_i = \sum_j a_{ji} = d$ for all i

- Captures both rings and sparse networks

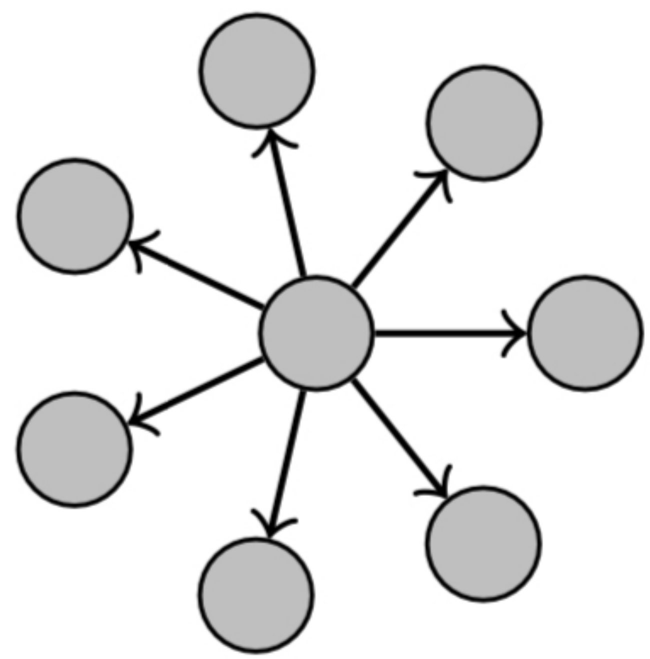




- In regular networks, sales are equal across all firms/sectors
- So by the previous discussion, microeconomic shocks wash out if n is sufficiently large

Asymmetric networks are fragile

- Asymmetry in importance of individual sectors gives rise to heterogeneity in sales and thus can cause micro-shocks to matter
- Extreme example: Star network



- The U.S. economy looks more like a star than a regular network

Baqaee and Fahri 2019

- **Key findings:**
  - Hulten's theorem generalizes to a wider class of models to a first-order
  - Additional (higher-order) moments matter for large shocks
- **Model:** Extend the canonical model to allow for general constant returns to scale production functions, preferences, and factor markets

$$y_i = z_i f_i(x_{i,1}, \ldots, x_{in}, l_{i1}, \ldots, l_{im})$$

- Household with homogeneous of degree 1 preferences, endowed with h_k units of the $k - th$ primary factor
- First Welfare Theorem: The decentralized equilibrium is efficient.
  - Can solve the planner's problem to solve for the equilibrium

Hulten's Theorem in Efficient Economies

- Planner's problem:

$$W = \max_{c_i, l_{ik}, x_{ij}} u(c_1, \ldots, c_n)$$

$$\text{s.t. } \forall i, c_i + \sum_{j=1}^{n} x_{ji} = z_i f_i(x_{i1}, \ldots, x_{in}, l_{i1}, \ldots, l_{im})$$

$$\forall k, \sum_{i=1}^{n} l_{ik} = h_k.$$

- First order condition for consumption

$$u'(c_i) = \eta_i$$

- The envelope theorem implies:

$$\frac{d \log W}{d \log z_i} = \frac{\eta_i y_i}{W}.$$

Hulten's theorem in efficient economies

- In the decentralized economy, the first order condition of the HH:

$$u'(c_i) = \phi p_i$$

- Further, for homogeneous utility functions, we know that

$$W = \phi GDP$$

- Together, these observations imply Hulten's theorem

$$\frac{d \log GDP}{d \log z_i} = \frac{d \log W}{d \log z_i} = \frac{\eta_i y_i}{W} = \frac{p_i y_i}{GDP} = \lambda_i$$

Non-linearities and shock propagation

- welfare effects can stem from two sources:
    1. Expansion in production possibility frontier (Technical Efficiency)
    2. Reallocation of resources across industries (Allocative Efficiency)
- in efficient economies, 2. is second-order
- second-order effects in the general (non-Cobb-Douglas) case

Theorem: $$\frac{d^2 \log GDP}{d \log z_i^2} = \frac{\lambda_i}{\sum_i \lambda_i} \sum_{j \neq i} \lambda_j \frac{d \log(\lambda_i/\lambda_j)}{d \log z_i} + \lambda_i \frac{\partial \log \sum_i \lambda_i}{\partial \log z_i}$$

- *i*ntuition: Cost shares $a_{ij} = p_j x_{ij} / p_i y_i$ and expenditure shares β_i are fixed in a Cobb-Douglas economy, so $\lambda_i = \sum_j l_{ji} \beta_j$ is fixed, too
- in general, Domar weights change, so second-order effects may be large

- Baqaee Fahri show that shocks to small but crucial industries („oil“) matter more than shocks to larger but less connected sectors („retail“)

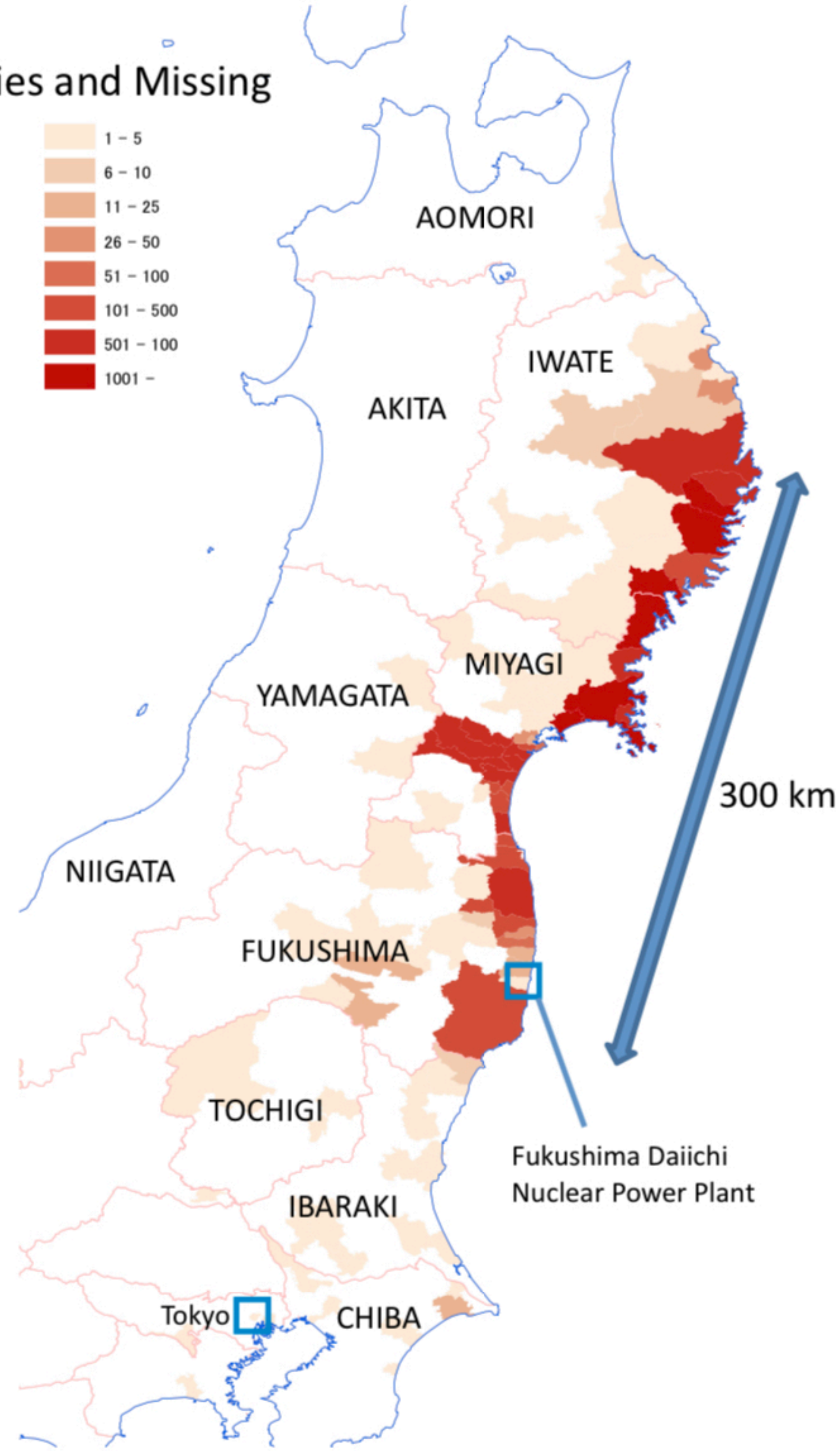
More on departures from Hulten's Theorem

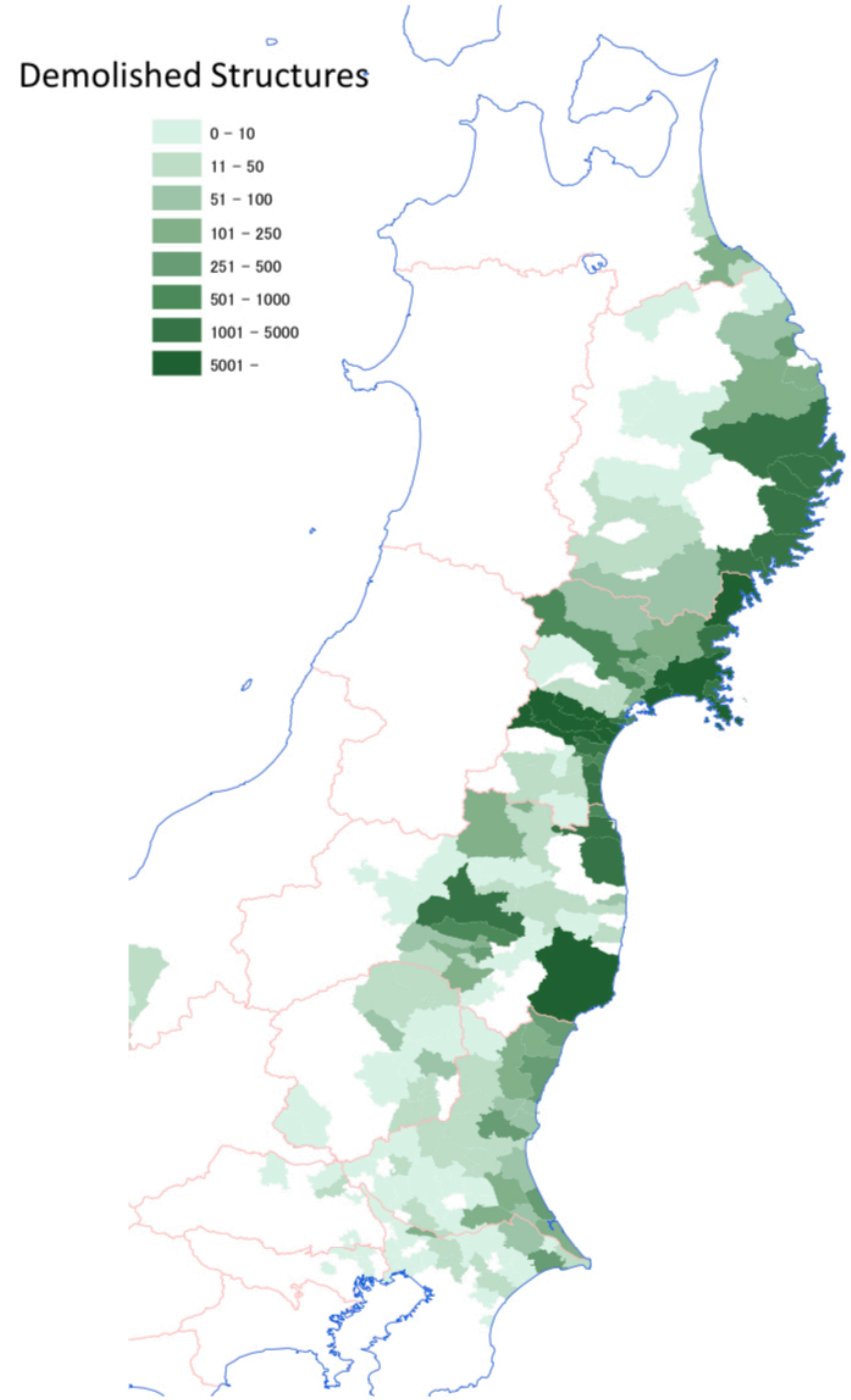
- Beyond focusing on higher-order moments, another way to go beyond Hulten's theorem are departures from efficiency
  - Distortions: Jones 2013, Bigio La'O 2020
  - Entry and scale economies: Baqaee 2018
- Next: Empirical studies providing evidence on the importance of firm-level shocks for aggregate outcomes

Carvalho Nirei Saite Tahbaz-Salehi 2020

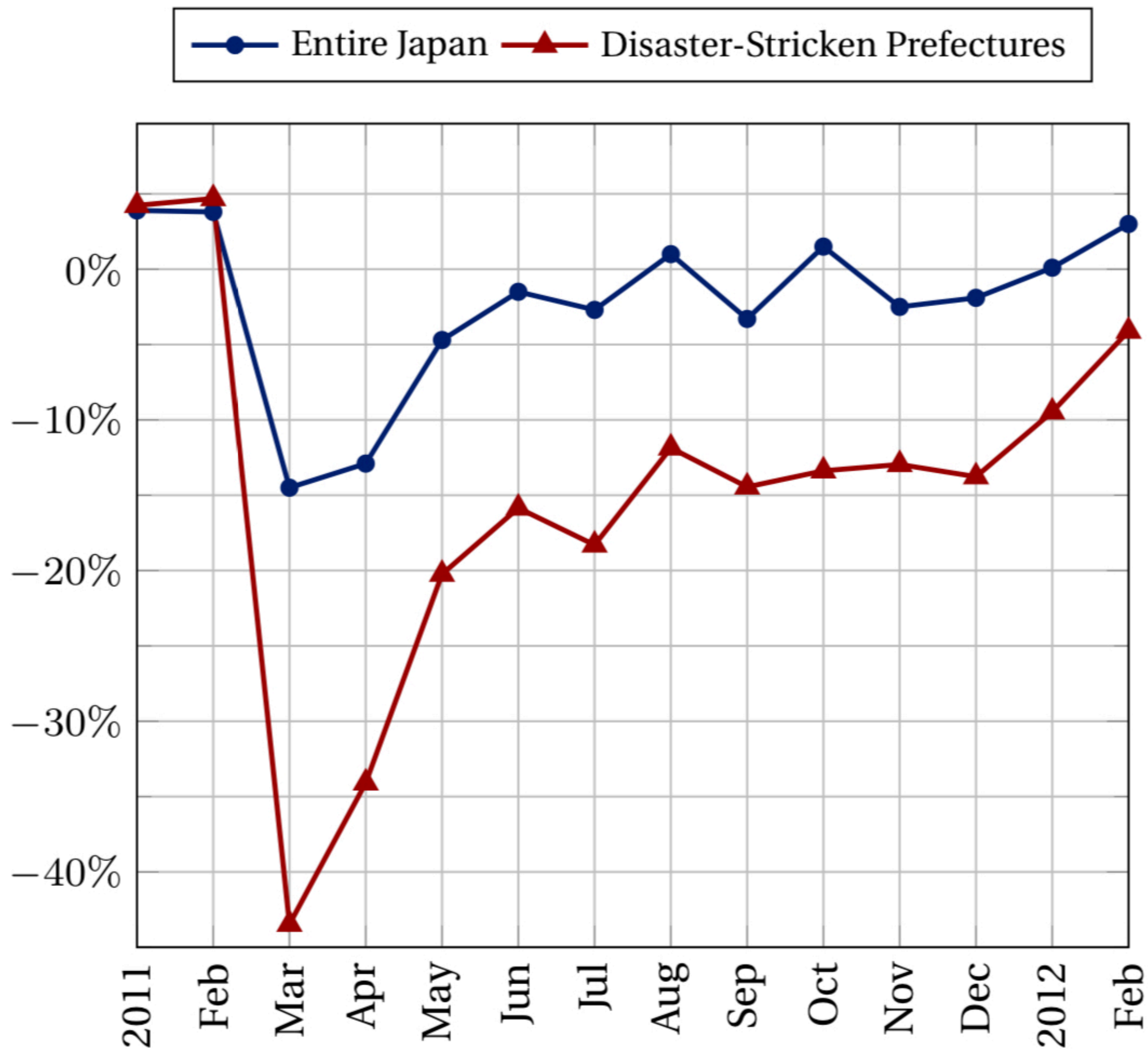
- How do adverse shocks spread through production networks?
- Can idiosyncratic shocks to individual firms cause aggregate fluctuations?
- **Approach**
  - Use the Great East Japan earthquake in 2011 as a “natural experiment” causing idiosyncratic shocks to firms
  - Provide direct micro-evidence of how these shocks propagate through the supply chain
  - Use a macro network model to estimates the GE effect

The Great East Japan Earthquake





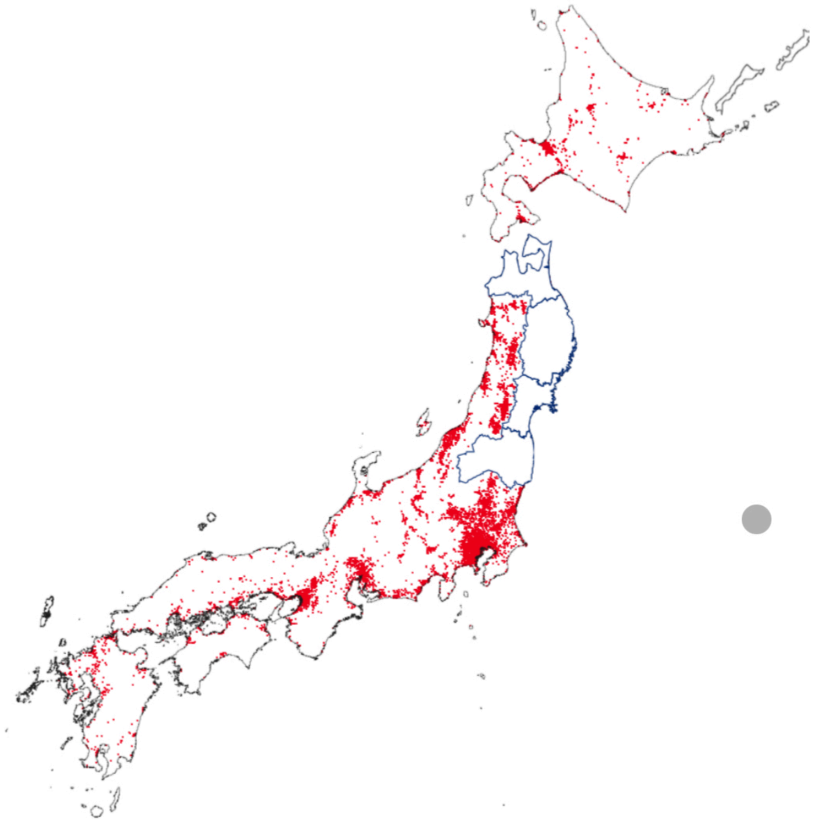
Effects on regional industrial production



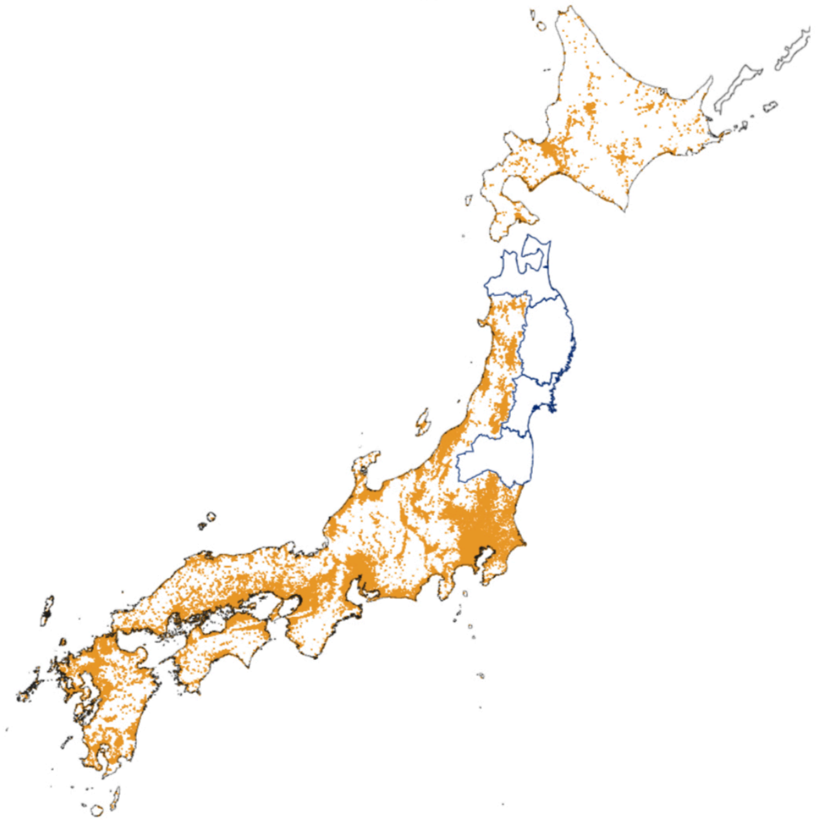
Data and Network Measures

- detailed firm-level data from a private credit reporting agency
  - Firms report a list of suppliers and costumers (binary, no intensive margin)
  - Sales and number of employees per firm
- "Disaster-area firms": Firms headquartered in regions that were under restricted access in the aftermath
- for 2010, construct measures of network distance:
  - Upstream distance 1: Direct supplier to a disaster area firm
  - Upstream distance 2: Direct supplier to a firm that is upstream distance 1...
  - Similarly for upstream (costumers)

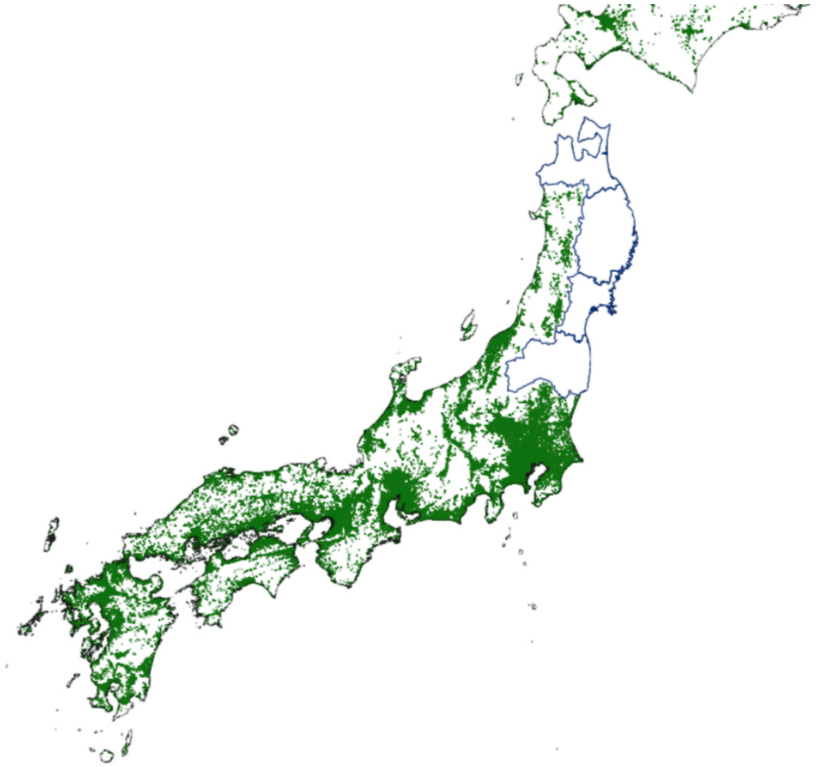
Network measures & disaster area



(a) distance 1



(b) distance 2



(c) distance 3

(d) distance 4

Pre-earthquake characteristics

	Disaster Area				Rest of Japan
	all firms	sorted by post-earthquake sales growth			
		bottom tercile	middle tercile	top tercile	
Log sales	11.39	11.37	11.48	11.31	11.54
	(1.67)	(1.64)	(1.69)	(1.68)	(1.77)
Log No. employees	1.83	1.74	1.88	1.88	1.84
	(1.27)	(1.24)	(1.35)	(1.22)	(1.31)
Age	27.41	27.94	27.87	26.41	29.00
	(15.48)	(15.45)	(15.80)	(15.13)	(16.54)
No. of suppliers	4.56	4.17	4.59	4.92	5.21
	(14.68)	(12.54)	(12.18)	(18.17)	(34.98)
No. of customers	4.56	3.02	4.77	5.82	5.17
	(30.38)	(6.36)	(37.70)	(36.12)	(36.13)
Customers' log sales	14.84	14.88	15.32	14.35	14.61
	(2.43)	(2.43)	(2.44)	(2.32)	(2.53)
Suppliers' log sales	14.63	14.76	14.63	14.52	14.87
	(2.39)	(2.45)	(2.45)	(2.30)	(2.56)

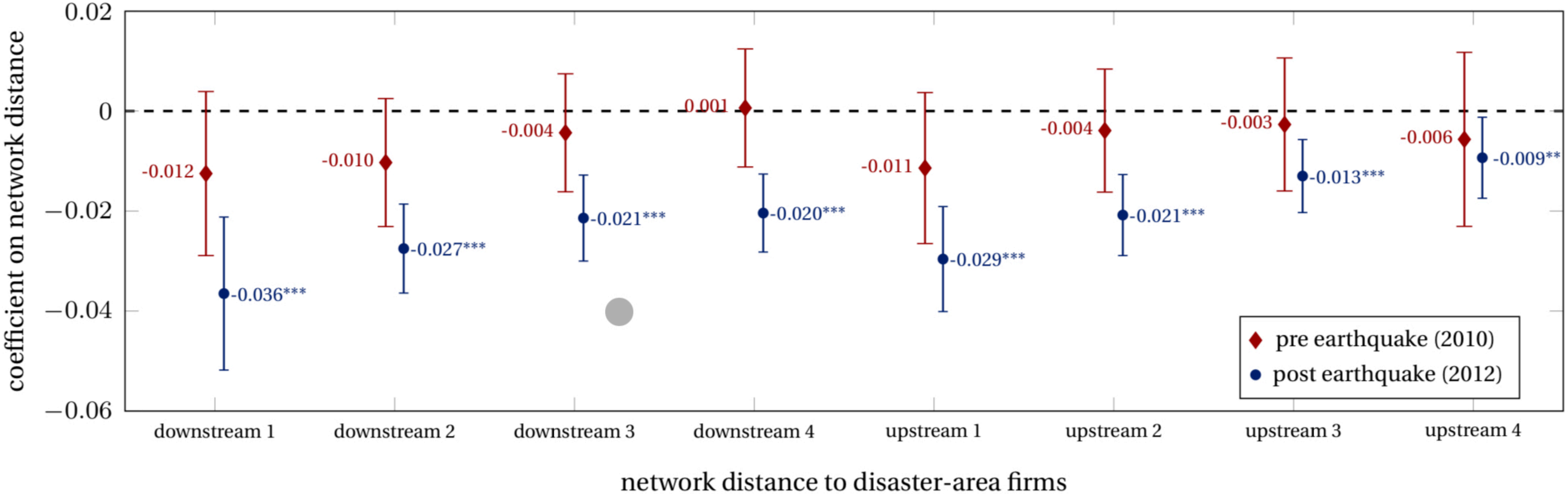
Empirical Strategy

- difference-in-difference approach

$$y_{ipst} = \ldots + \sum_{k=1}^{f} \sum_{\tau \neq 2011} \beta_{k,\tau}^{\text{down}} \times Downstream_i^{(k)} \times year_\tau + \sum_{k=1}^{f} \sum_{\tau \neq 2011} \beta_{k,\tau}^{\text{up}} \times Upstream_i^{(k)} \times year_\tau + \ldots$$

- y_{ipst}: Log-sales of firm i in industry s and with headquarters in prefecture p
- downstream: 1 if firm is a distance (k) costumer of disaster-area firms
- upstream: 1 if firm is a distance (k) supplier to distaster-area firms
- control group: All firms that were 5 or more supply chain links away from disaster-area firms
- identification: Conditional on observables, the supply-chain distance to distaster area firms is orthogonal to any unobservable characteristics that affect post earthquake output

Results



0.02
0
−0.02
−0.04
−0.06
coefficient on network distance
-0.012
-0.036***
-0.010
-0.027***
-0.004
-0.021***
0.001
-0.020***
-0.011
-0.029***
-0.004
-0.021***
-0.003
-0.013***
-0.006
-0.009**
pre earthquake (2010)
post earthquake (2012)
downstream 1
downstream 2
downstream 3
downstream 4
upstream 1
upstream 2
upstream 3
upstream 4
network distance to disaster-area firms

Robustness

- Definition of the disaster area
  - Disaster-area= all regions that got flooded
- Single-plant firms
  - Firms with headquarters outside of disaster-area and linkages to disaster-area firms might be multi-plant with plants in the disaster area
- Only firms located in western japan
  - Tsunami caused long period of rolling power outages
  - Firms close to the disaster area might be more linked, but also more exposed to subsequent power outages
  - Due to idiosyncrasies of the Japanese electcity grid, electricity disruptions only affected eastern Japan

Theoretical Framework

- Goal: Quantify the macroeconomic impact of the disaster
- Diff-in-diff provides a PE effect, so construct a model for GE effects
- Model is as in the Canonical framework, but i denotes firms rather than industries
- production functions are nested-CES instead of Cobb-Douglas (why?)

$$y_i = \left[\chi(1-\mu)^{1/\sigma}\left((z_i k_i)^\alpha l_i^{1-\alpha}\right)^{(\sigma-1)/\sigma} + \mu^{1/\sigma} M_i^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$$

$$M_i = \left(\sum_{j=1}^{n} a_{ij}^{1/\xi} x_{ij}^{(\xi-1)/\xi}\right)^{\xi/(\xi-1}$$

- z_i is value-added productivity rather than total factor productivity

Propagation of Shocks

Proposition: The impact of a shock to firm j on the sales share λ_i of firm i is to a first order given by:

Impact on substitution between primary and intermediate inputs

$$\frac{d \log \lambda_i}{d \log z_j} = (\sigma - 1) \sum_{h=1}^{n} \alpha\mu(1-\mu)\frac{\lambda_h}{\lambda_i} \left[(\sum_{r=1}^{n} a_{hr} l_{rj})(\sum_{s=1}^{n} a_{hs} l_{si}) - l_{hj} \sum_{r=1}^{n} a_{hr} l_{ri} \right]$$

$$+ (\xi - 1) \sum_{h=1}^{n} \alpha\mu(1-\mu)\frac{\lambda_h}{\lambda_i} \left[\sum_{r=1}^{n} a_{hr} l_{rj} l_{ri} - (\sum_{r=1}^{n} a_{hr} l_{rj})(\sum_{s=1}^{n} a_{ks} l_{si}) \right]$$

Impact on substitution between various intermediate inputs

- l_{ij} : element (i, j) of the Leontief inverse $L = (I - \mu A)^{-1}$
- They use this equation to estimate σ and ξ (using I-0 tables to construct A)

Aggregation

- Changes in GDP to a second order

Hulten's Theorem

$$d \ln GDP = \alpha(1-\mu)1'\left(\frac{\Lambda + \Lambda^*}{2}\right) d \ln z$$

Non-linearities

$$+\frac{1}{2}\alpha(\mu(1-\mu)(\sigma-1)d \ln z'\Lambda(I-A)Ld \ln z$$

- $d \ln z_i = -0.32$ for disaster area firms (government estimate)

Findings

- The distaster reduced GDP growth by 0.47 percentage points
  - Average GDP growth in the decade before: 0.6%
- Removing I-O linkages
  - Set $a_{ij} = a_{ji} = 0$ whenever one firm is in disaster area and one is not
  - Then, the disaster would have reduced GDP growth by 0.21 percentage points
  - Highlights the importance of non-linear propagation as the network only matters for the second-order term on the previous slide

di Giovanni Levchenko Mejean (ECMA 2014)

- **Question:**
  - What is the role of individual firms in generating aggregate fluctuations of French sales growth?
- **Strategy:** Micro-to-macro accounting
  - Decompose a firm's annual sales growth into i) firm-destination shock ("micro"), ii) sector-destination shock ("macro")
  - Use estimates to measure the contribution of the firm component to aggregate fluctuations (variance of aggregate sales growth)
  - Relate the contribution of the firm component to the firm size concentration and interconnection across firms

Overview of Results

- More than 90% of firm-level sales growth accounted for by firm component
- Around 80% of aggregate fluctuations accounted for by firm component
- Contribution of firm component is larger for fluctuations in aggregate exports
- Volatility of the firm-specific component is correlated with the distribution of firm sizes and the magnitude of I-O linkages

Aggregate Growth

- Total aggregate sales by all French firms

$$X_t = \sum_{f,n} x_{fnt}$$

where x_{fnt} is firm f's sales to destination n at time t

- Up to a first order, aggregate sales growth is

$$\gamma_{At} = \sum_{f,n} \omega_{fnt_0} \gamma_{fnt}$$

where

- ω_{fnt_o}= share of firm f's sales in aggregate sales in base period
- γ_{fnt}: growth rate between t and $t-1$

Firm-level growth

- Decompose firm-level growth into

$$\gamma_{nft} = \delta_{jnt} + \varepsilon_{fnt}$$

δ_{jnt}: sector-destination-year specific shock (“macro”)

ε_{fnt}: firm-destination-year specific shock (“micro”)

- These decompositions can be motivated by a multi-sector heterogeneous firms model in the spirit of Melitz (2003)
- This can be estimated, year-by-year and destination-by-destination, using OLS with fixed effects to identify sector-destination shocks

Variance decomposition

- The variance of aggregate growth is

$$\sigma^2_{At0} = var\left(\sum_{j,n} \omega_{jnt_0}\delta_{jnt}\right) + var\left(\sum_{j,n} \omega_{fnt_0}\epsilon_{fnt}\right) + Cov_t$$

Macro Volatility **Firm Volatility**

- Goal is to study to what extent the "firm" component "explains" aggregate fluctuations
    - Same strategy as in development accounting literature
    - Compute ratio between firm component and variance of aggregate growth: If large, firms are important.

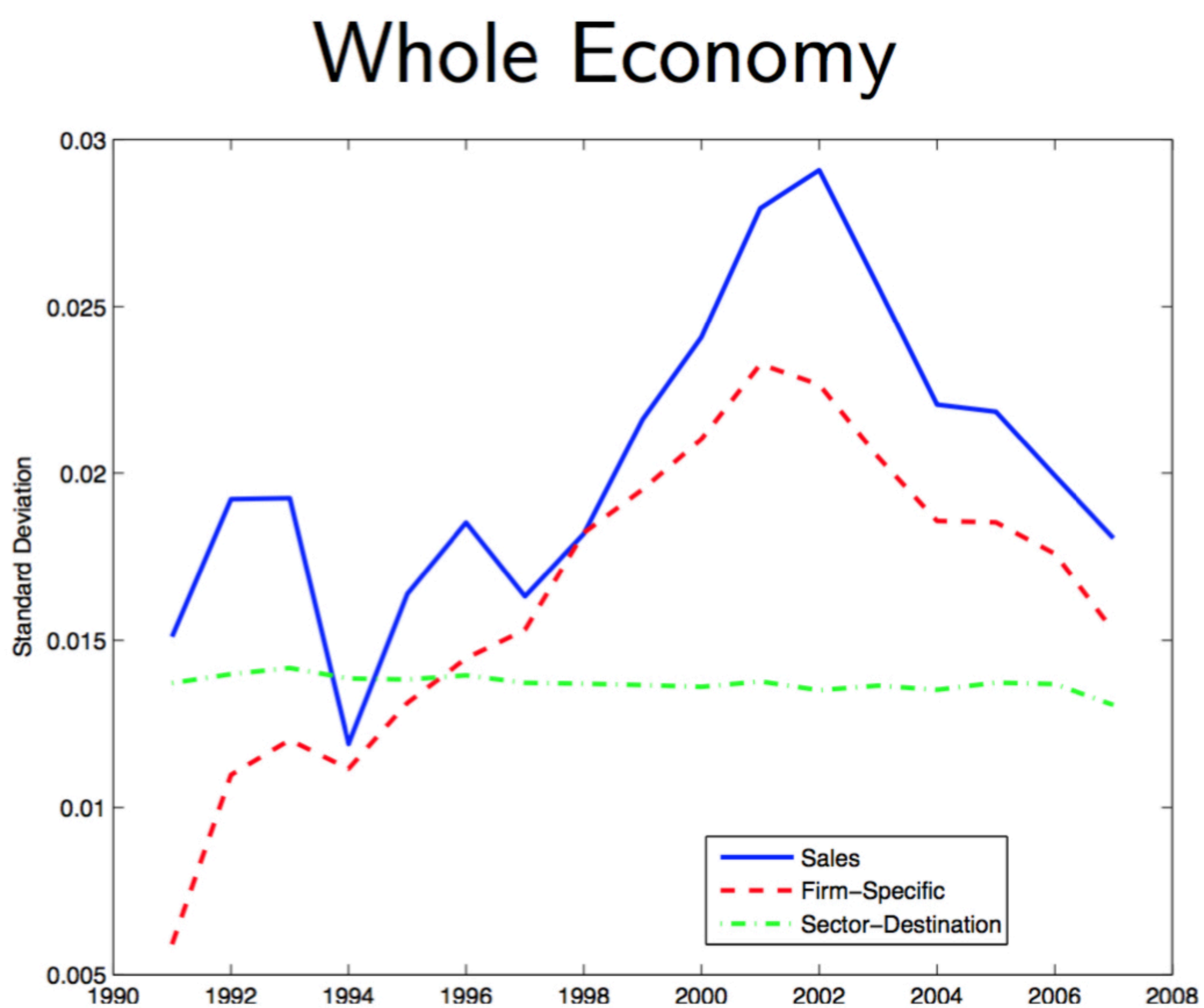
Data description

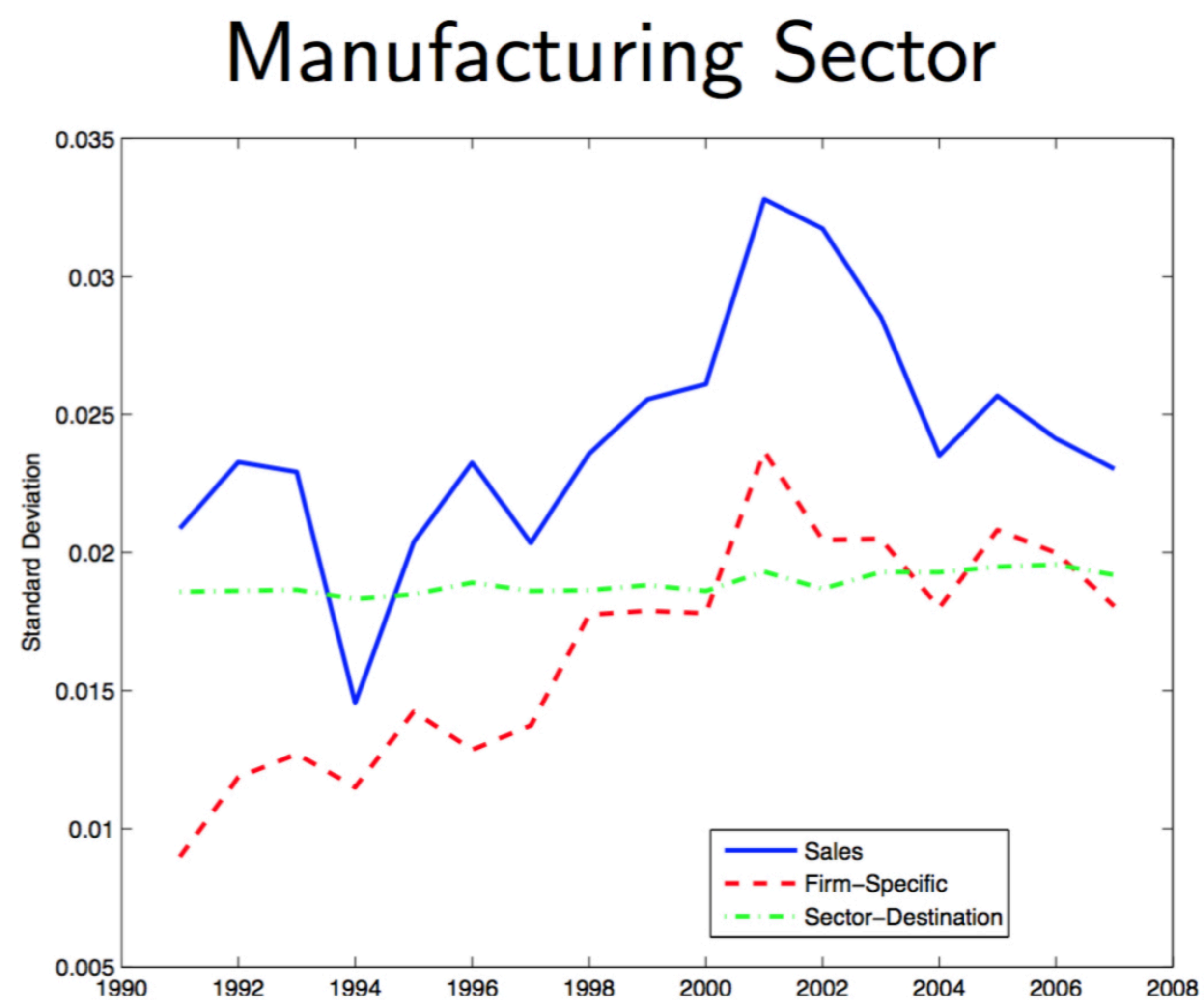
- Firm-level domestic and export sales data for the universe of French firms over 1990-2007
- Merge two large datasets
  - Fiscal administration: firm tax forms
  - Costums: firm-destination exports

Aggregate Volatility Accounting

	I. Total Sales			
	Whole Economy		*Manufacturing Sector*	
	(1)	(2)	(3)	(4)
	St. Dev.	Relative SD	St. Dev.	Relative SD
Actual	0.0206	1.0000	0.0244	1.0000
Firm-Specific	0.0165	0.8010	0.0168	0.6885
Sector-Destination	0.0109	0.5291	0.0157	0.6434
	II. Domestic Sales			
	Whole Economy		*Manufacturing Sector*	
	(1)	(2)	(3)	(4)
	St. Dev.	Relative SD	St. Dev.	Relative SD
Actual	0.0196	1.0000	0.0231	1.0000
Firm-Specific	0.0154	0.7857	0.0151	0.6537
Sector-Destination	0.0112	0.5714	0.0167	0.7229
	III. Export Sales			
	Whole Economy		*Manufacturing Sector*	
	(1)	(2)	(3)	(4)
	St. Dev.	Relative SD	St. Dev.	Relative SD
Actual	0.0361	1.0000	0.0374	1.0000
Firm-Specific	0.0304	0.8421	0.0287	0.7674
Sector-Destination	0.0129	0.3573	0.0153	0.4091

Contribution of firms increases over time





What determines the Firm-Component?

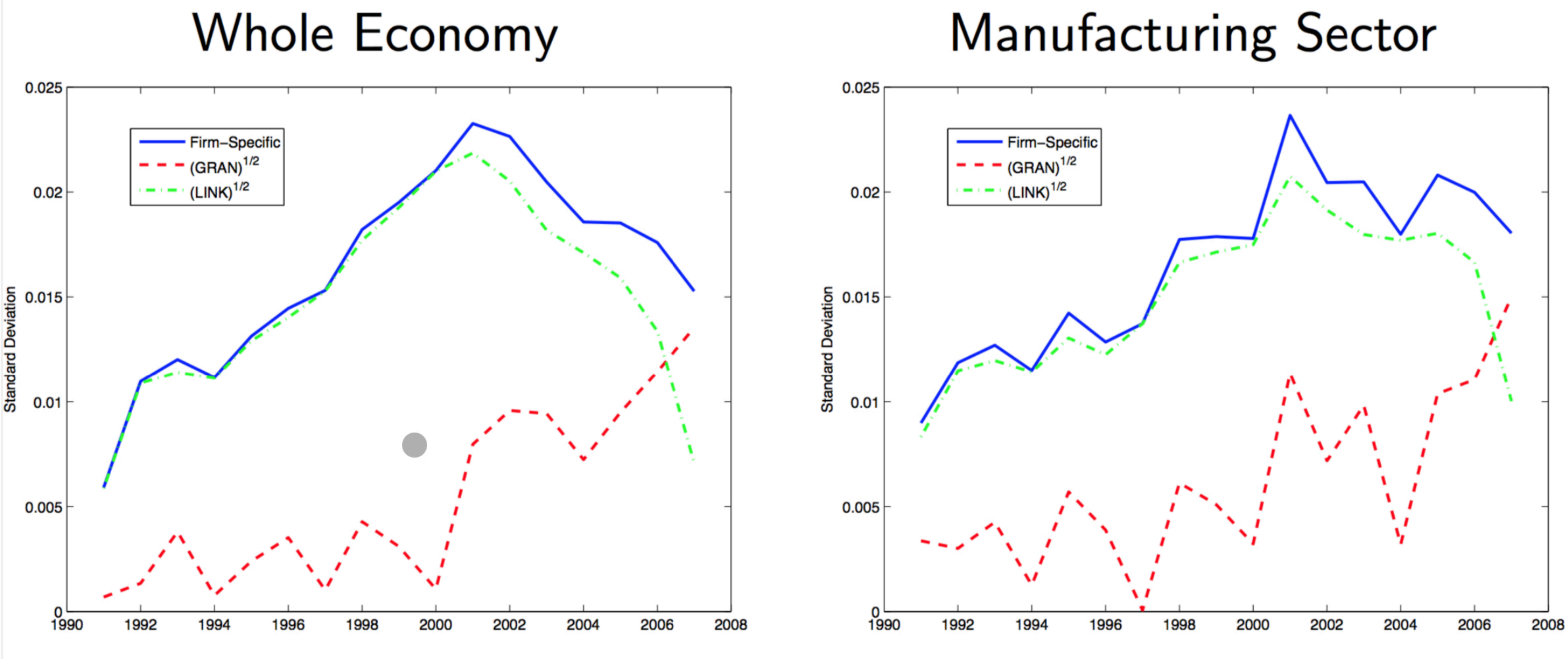
- We can write:

$$\sigma^2_{Ft_0} = var(\sum_{f,n} \omega_{fnt_0} \epsilon_{fnt})$$

$$= \underbrace{\sum_{i,n} \omega^2_{fnt_0} var(\epsilon_{fnt})}_{\text{GRAN}} + \underbrace{\sum_{g \neq f, m \neq n} \sum_{f,n} \omega_{gmt_0} \omega_{fnt_0} cov(\epsilon_{gmt}, \epsilon_{fnt})}_{\text{LINK}}$$

- With i.i.d. Shocks and symmetric firms, we would expect GRAN=LINK=0
- Which departures matter more in practice?

Granularity and linkages



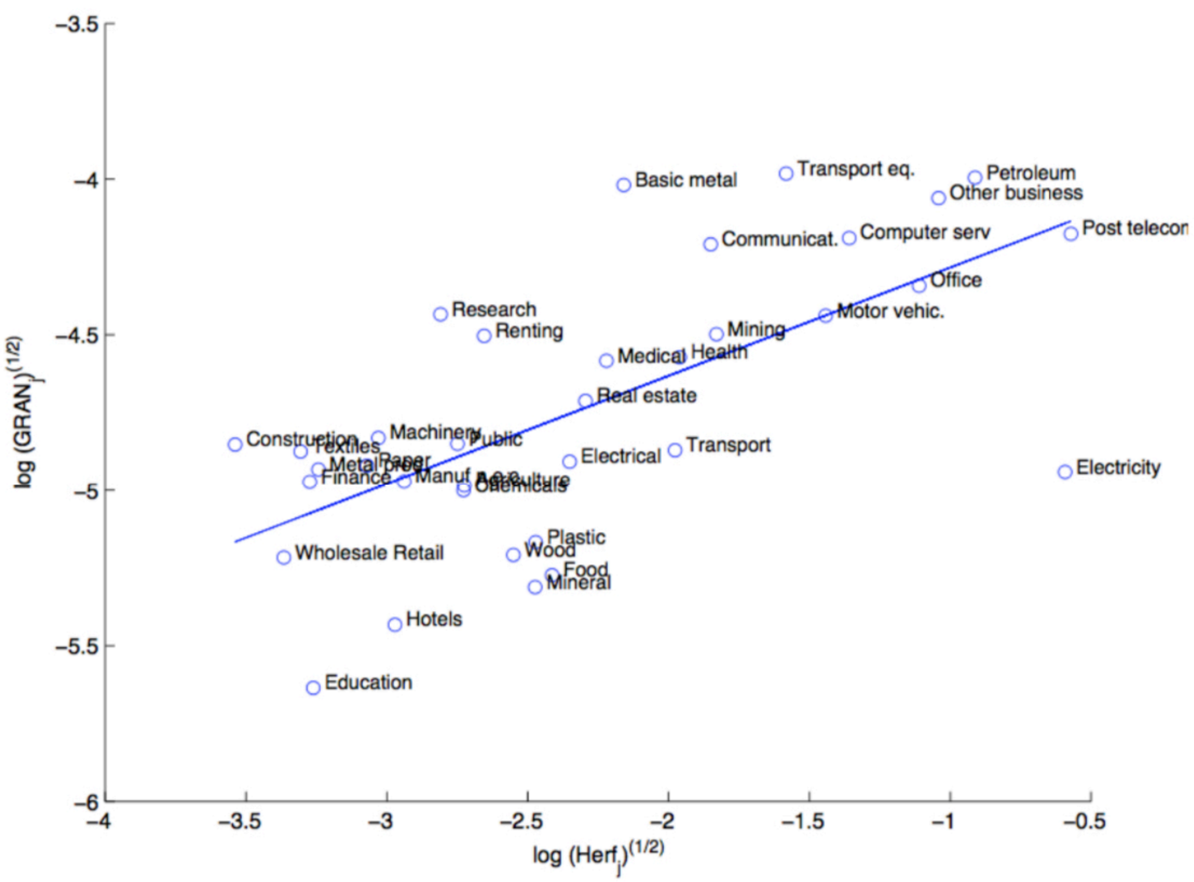
- Linkages explain most of firm-level volatility
- Granular contribution is rising over time

Granularity across sectors

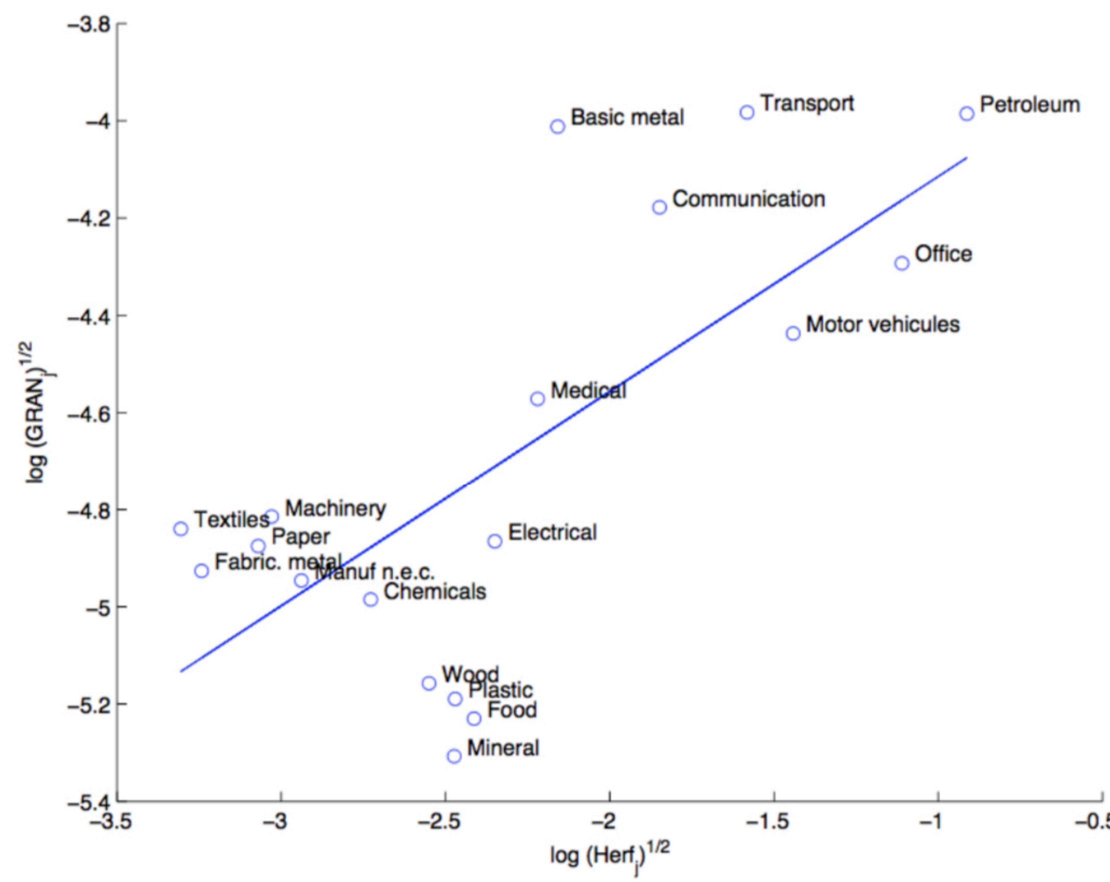
- More concentrated sectors should display more granularity

$$GRAN = \sum_j GRAN_j \text{ and } GRAN_j = \sum_{(f,n)\in j} \omega^2_{fn,t-1} var(\varepsilon_{fnt}) = \sigma^2 HERF_{j,t-1}$$

Whole Economy



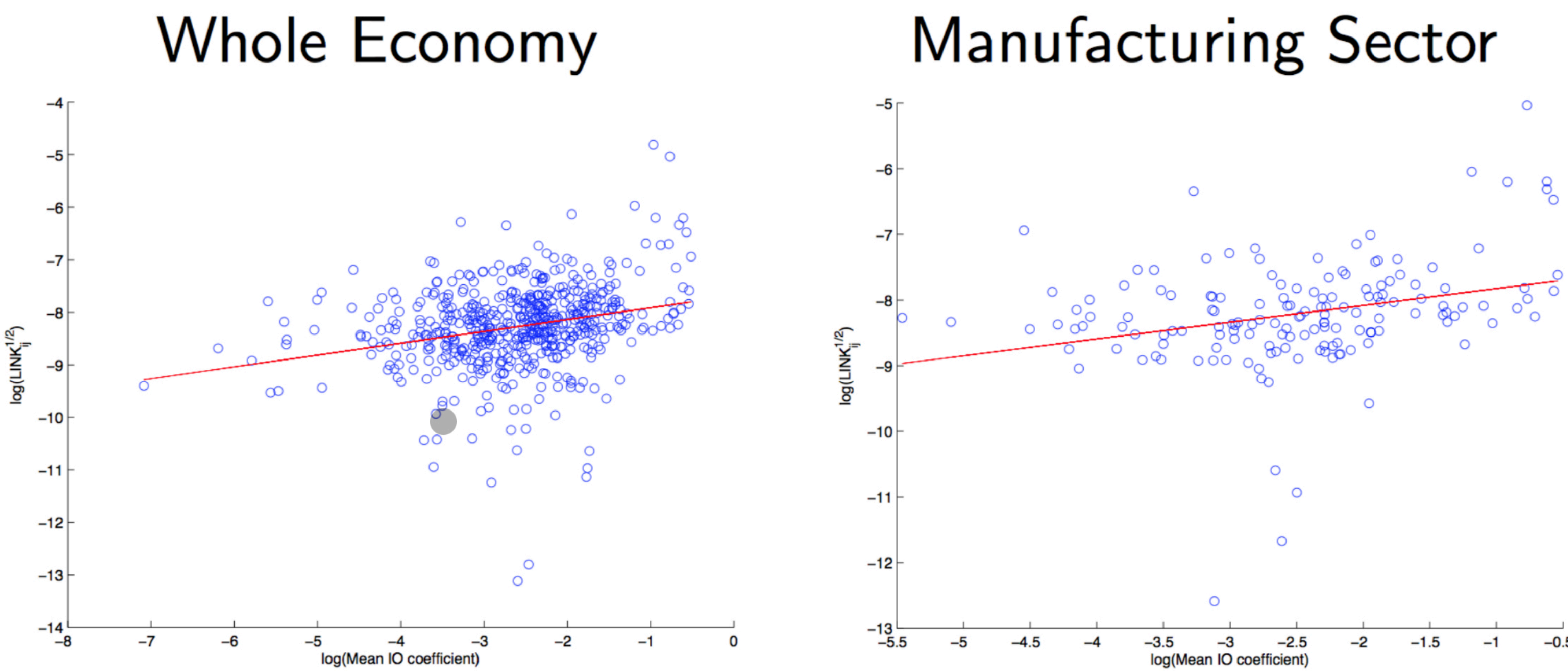
Manufacturing Sector



Linkages across sectors

$$LINK_{ij} = \sum_{(f,n)\in i} \sum_{(g,m)\in j} \omega_{f,n,t-1}\omega_{g,m,t-1} cov(\varepsilon_{f,n,t}, \varepsilon_{g,m,t})$$

- Are linkages related to mean IO coefficients $(a_{ij} + a_{ji})/2$?



Caliendo, Parro, Rossi-Hansberg Sarte

- Aggregate fluctuations are the result of a wide variety of disaggregated TFP changes
  - Sectoral: process or product innovation
  - Regional: Natural disasters, changes in local regulations
  - Sectoral and regional: corporate bankruptcy or bailouts
- Quantitative model adding trade and migration to the canonical model:
  - Input-output + inter-regional trade and migration linkages
  - Differences in regional and sectoral TFP, local factors and geography (fixed supply of factors)
- Key finding: Aggregate GDP elasticity to local productivity changes varies substantially:
  - 1.6 in NY, 1.3 in CA, but only 0.89 in FL and 0.34 in WI

The model

- Multi-sector-region version of Eaton-Kortum (2002) with
  - Labor mobility across regions
  - Roundabout production structure
- N regions, J sectors, two factors
  - Labor L_n^j: mobile across regions and sectors
  - Land and structures H_n: Fixed across regions, mobile across sectors

Households

- An agent in region n maximizes Cobb-Douglas utility and receives income from labor, payments from a national housing portfolio, and local rents

$$v_n = \max_{\{C_n^j\}} \prod_j \left(C_n^j\right)^{\alpha^j}$$

$$\text{s.t. } \sum_j P_n^j C_n^j = w_n + \frac{\sum_i \iota_i r_i H_i}{\sum_i L_i} + (1 - \iota_n)\frac{r_n H_n}{L_n}$$

- In equilibrium households are indifferent between living in any region so

$$v_n = \frac{I_n}{P_n} = U$$

where P_n is the ideal price index in region n

Production

- As in Eaton-Kortum, in region n and sector j there is a unit continuum of goods
  - Consumers aggregate them according to a CES aggregator to obtain the consumption index
  - Firms aggregate them in the usage of intermediate goods according to the same CES aggregator
- The production function of a variety of a variety with idiosyncratic productivity z produced in region n in sector s is given by:

$$q_n^j(z) = z\left[T_n^j h_n^j(z)^{\beta_n} l_n^j(z)^{1-\beta_n}\right]^{\gamma_n^j} \prod_{k=1}^{J} M_n^{jk}(z)^{\gamma_n^{jk}}$$

  - T_n^j: value-added productivity
  - M_n^{jk}: the CES aggregate over the unit continuum of goods

Unit cost and prices

- In Eaton-Kortum, a variety is sourced from the cheapest origin
- Here, we do the same thing (origins = regions), but we need to write down unit cost
- Cost of the input bundle, given the Cobb-Douglas production function are given by

$$x_n^j = B_n^j \left[r_n^{\beta_n} w_n^{1-\beta_n} \right]^{\gamma_n^j} \prod_k \left(P_n^k \right)^{\gamma_n^{jk}}$$

- Everyone - firms and consumers - picks the cheapest place to source a variety

$$p_n^j(z^j) = \min_i \left\{ \frac{\kappa_{ni}^j x_i^j}{z_i^j} \left(T_i^j \right)^{-\gamma_i^j} \right\}$$

- Now, assume that z are drawn i.i.d. from a Frechet distribution with dispersion parameter θ^j, and we are back in Eaton-Kortum business

Aggregation: Goods markets

- As in Eaton-Kortum, the expenditure share of region i on i sector $k's$ goods purchased from region n is given by:

$$\pi_{in}^{k} = \frac{\left[x_n^k \kappa_{in}^k\right]^{-\theta^k} \left(T_n^k\right)^{\theta^k \gamma_n^k}}{\sum_m \left[x_m^k \kappa_{im}^k\right]^{-\theta^k} \left(T_m^k\right)^{\theta^k \gamma_m^k}}$$

- Ideal price index for aggregate goods from sector k in region i:

$$\left(P_i^k\right)^{-\theta^k} = \zeta_i^k \sum_m \left[x_m^k \kappa_{im}^k\right]^{-\theta^k} \left(T_m^k\right)^{\theta^k \gamma_m^k}$$

- Total revenue accounting for sector k in region i is given by:

$$X_i^k = \sum_n \pi_{ni}^k \alpha^k I_n L_n + \sum_n \sum_j \pi_{ni}^k \gamma_n^{jk} X_n^j$$

(This equation takes a different but essentially equivalent form in the paper)

Labor market clearing

- Populations across regions add up to total labor supply

$$\sum_n L_n = L$$

- Firm optimization (Cobb-Douglas + same labor shares across all sectors in a region) implies:

$$w_n L_n = \frac{1-\beta_n}{\beta_n} r_n H_n$$

- Free mobility ($I_n = P_n U$) implies: $L_n = H_n \zeta_n$, where

$$L_n = H_n \left[\frac{\omega_n}{P_n U + \iota_n r_n H_n / L_n - \frac{\sum_i \iota_i r_i H_i}{L}} \right]^{1/\beta_n} \quad \text{where } \omega_n \equiv \left(\frac{r_n}{\beta_n}\right)_n^{\beta} \left(\frac{w_n}{1-\beta_n}\right)^{1-\beta_n}$$

- So the labor input in each region is pinned down by:

$$L_n = \frac{L_n}{L} L = \frac{H_n \zeta_n}{\sum_i H_n \zeta_i} L$$

Equilibrium

- Previous conditions pin (as a function of w_n) jointly pin down the rental rate and labor supply in each location
- To pin down wages: Labor income = labor payments

$$w_n L_n = (1 - \beta_n) \sum_j \gamma_n^j X_n^j$$

- Writing the equilibrium conditions in terms of demand for factors (rather than balanced trade) is sometimes convenient to prove existence and uniqueness in equilibria

Solution algorithm for these types of models

- Guess wages for all regions (outer loop)
  - Compute labor allocations and rental rates consistent with firm optimization and free mobility (a fixed point)
  - Given wages and rental rates, compute unit cost and price indices (another fixed point)
  - Given unit cost compute matrix of trade shares
  - Given matrix of trade shares and incomes, compute sales in each industry (a fixed point)
  - Given the sales, update wages from the factor market clearing condition
  - Compare to initial guess, update guess and repeat until convergence

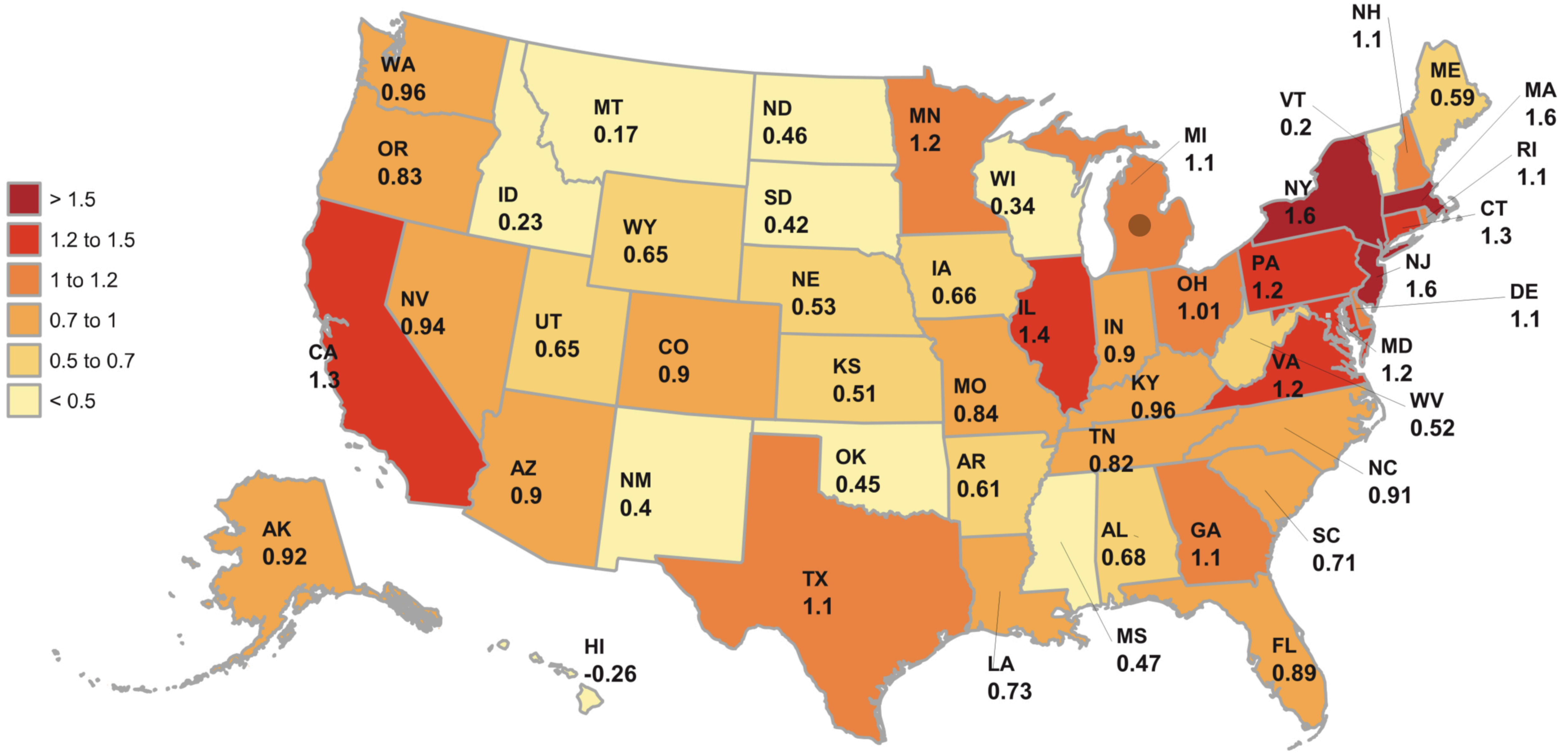
Calibration and Quantitative exercise

- To calibrate the model, they use a variety of data sources
  - BEA value added shares + national IO tables to compute internal I-O matrix (with proportionality assumptions)
  - BEA for employment across states
  - I_n is value added in each state in 2007
  - Inter-regional trade from Commodity Flow Survey
- Baseline exercise: Increase productivities T_n^j by 10% for all sectors, and compute GDP elasticities:

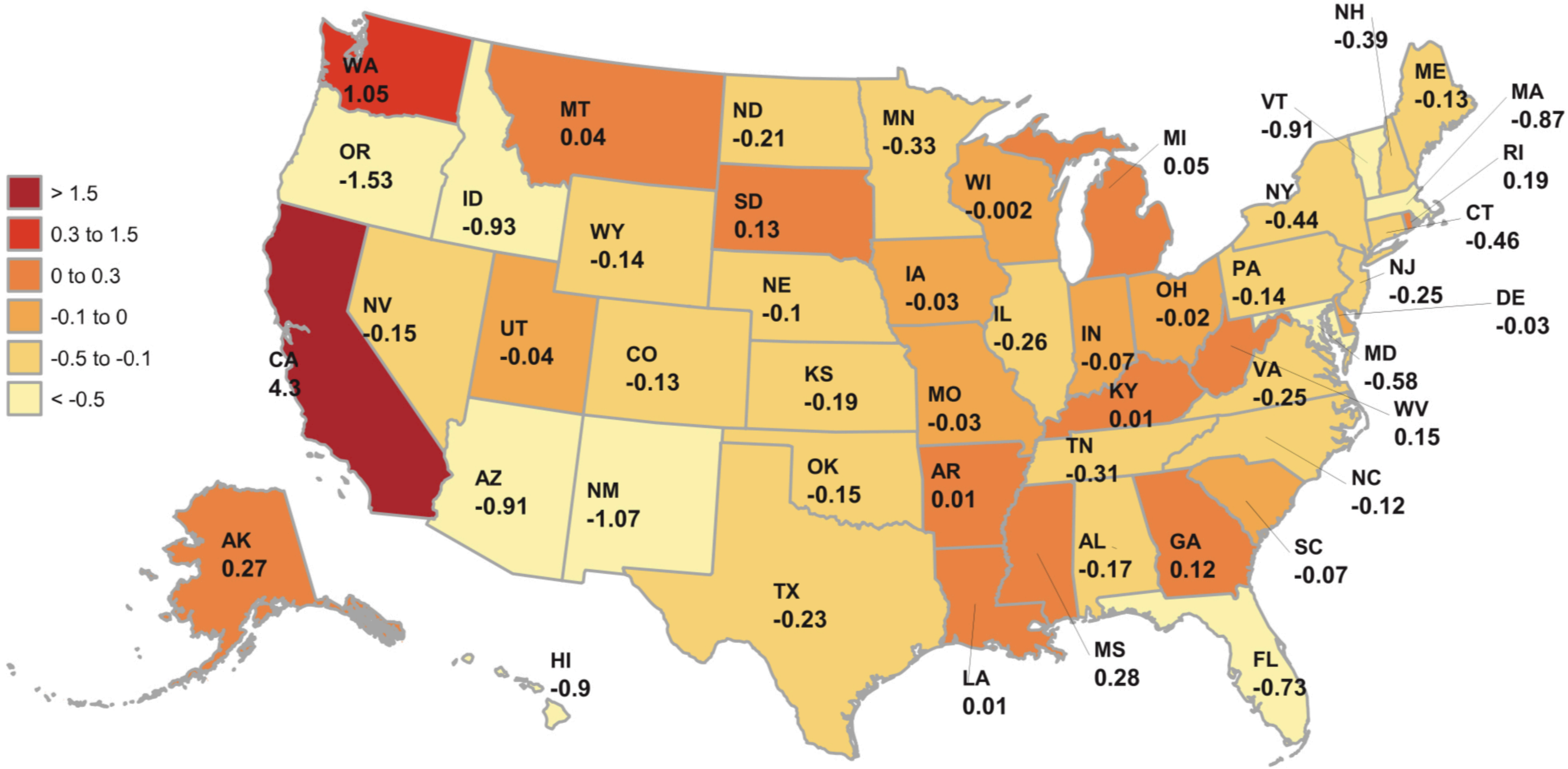
$$GDP\ elast. = \frac{d \log GDP}{(w_n L_n / L) d \log T_n}$$

- Also in the paper: „measured total factor productivity“ elasticities and welfare elasticities

Results: Aggregate GDP elasticities



A 31% increase in Computer and electronics TFP in CA - effect on state level GDP



More

- empirical studies on production network propagation
  - Barrow Sauvagnan 2016: Use natural disasters in the U.S. to study how exogenous shocks to individual firms propagate downstream and upstream
  - Demir et al 2018: The propagation of financial shocks to liquidity-constrained firms
  - Ozdagli Weber 2017: Propagation of monetary policy shocks
- Existing quantitative models (with/without migration) rely on C-D everywhere
  - More realistic substitution patterns in technology/preferences?
- Endogenous production networks: How do firms match up with suppliers?
  - Attempts: Lim 2018, Oberfield 2018, Acemoglu and Azar 2020
  - No comprehensive framework exists thus far
- What about frameworks where some suppliers are substitable, but others are not at all? Nothing so far...