

# A Matter of Taste: A unified approach to modeling monopsony

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## **Abstract**

I establish a fundamental equivalence between search-theoretic and preference-based approaches to modeling monopsony. When time discounting vanishes, the employment distribution from optimal job search in frictional monopsony models mirrors the labor allocation chosen by an agent with non-homothetic preferences, with the mean-min wage ratio emerging as a sufficient statistic for how wage inequality shapes employer substitutability. This equivalence challenges the conventional distinction between "frictional" and "taste-driven" monopsony, and yields practical insights for measurement and policy design. As an application, I derive and quantify a formula for welfare gains from employer entry that requires standard labor market statistics in search-based models but demands residual elasticity estimates in taste-based models. This formula reveals that job creation can reduce welfare below a critical unemployment threshold that rises with on-the-job search efficiency, with gains showing counter-cyclical patterns that peak during labor market downturns.

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# 1 Introduction

Labor markets convert wage offers into employment allocations. Two different approaches have emerged to model this transformation under imperfect competition. On one hand, search-theoretic models emphasize the role of frictions—such as random contacts and separation rates—that slow job transitions, generate wage dispersion, and contribute to involuntary unemployment (Burdett 1978, Burdett & Mortensen 1998). On the other hand, preference-based models attribute wage dispersion and unemployment to idiosyncratic tastes and voluntary choices (e.g., Card *et al.* 2018, Berger *et al.* 2022).<sup>1</sup>

This paper bridges these seemingly disparate frameworks by demonstrating that a broad class of wage-posting models with on-the-job search yields firm employment distributions identical to those produced by a representative agent with non-homothetic preferences over jobs. I show that this equivalence arises naturally from the mechanics of optimal job search, independent of how wage offers are generated on the labor demand side.

At the heart of my analysis is the insight that search frictions may limit worker mobility in precisely the same way as employer differentiation in tastes—both normatively and positively.<sup>2</sup> The equivalence result reveals that job ladder models generally spell non-homotheticities in firm-level labor supply because aggregate welfare and the reservation wage do not scale proportionally with wages. Although random utility models typically assume homothetic preferences with no income effects, I demonstrate that each framework can be adjusted to nest within the unified formulation. In essence, what appear to be fundamentally different modeling approaches are merely alternative representations of the same limitations to worker mobility.

This unified framework not only reconciles two paradigms of the modern monopsony literature but also provides a powerful tool for labor market analysis, clarifying the importance labor supply structure and its microfoundations for aggregate market adjustment, welfare and policy design. As an illustration, I derive a parsimonious formula for measuring the welfare gains from job creation—a "love-for-

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<sup>1</sup>Manning (2021) refers to this class as “New Classical Monopsony” models, alluding to their conceptual proximity to Robinson’s initial conceptualization of monopsony from 1933. Other authors, such as Berger *et al.* (2022, 2024), have used the term “neoclassical” to describe these models.

<sup>2</sup>In taste-based models, limitations to worker mobility arise from information frictions rendering workers’ idiosyncratic tastes for non-wage amenities unobservable to employers (see, e.g., the discussion in Lamadon *et al.* 2022). My contribution is to show that the aggregate implications of these information frictions are indistinguishable from the positive and normative effects of search frictions.

variety" effect—that holds under search-theoretic and preference-based perspectives. Under the search interpretation, the formula relies on standard labor market statistics, whereas the preference-based view requires estimation of residual elasticities. Applied to U.S. data (1999–2024), this measure reveals new quantitative insights into the normative effects of job-ladder cyclicalities.

I formalize these insights within a job-ladder model of random on-the-job search with firm-specific contact rates, following the canonical formulation by Burdett (1978) and Burdett & Mortensen (1998). I prove that when the discount rate becomes negligible compared to the rate at which job seekers encounter new job opportunities,<sup>3</sup> the mapping from wage offers to employment can be summarized by a labor supply system

$$\frac{n_\omega w_\omega}{Y} = \frac{w_\omega \ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right)}{\int_\Omega w_\omega \ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right) d\omega},$$

where each firm  $\omega$ 's employment  $n_\omega$  depends on its wage offer  $w_\omega$ , total employment income  $Y$  and a wage aggregator  $\Lambda$  that adjusts to satisfy the budget constraint. Importantly, the residual labor supply curves  $\ell_\omega$  as well as the function  $K$  are all defined over wages and are parametrized by the underlying search technology.

I then show that this labor supply system can be rationalized by a non-homothetic preference over firm-level labor allocations in which worker mobility restrictions arise from taste-based employer differentiation.<sup>4</sup> In this preference, the aggregator  $\Lambda$  captures the elasticity of utility with respect to total employment income. Importantly, in search models this elasticity typically deviates from one because the reservation wage does not scale proportionally with wages. Consequently, the labor supply system for firms exhibits income effects mediated by the wage shifter  $K(\Lambda)$ . In the search model the wage shifter corresponds to the mean-min ratio—the ratio of the average wage to the reservation wage. My results show that  $K(\Lambda)$  satisfies an envelope condition that ensures the mutual consistency of cross-sectional labor allocations, optimal job search behavior and the representative agent's resource constraint at any level of income. This

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<sup>3</sup>This is akin to the class of "timeless" equilibria commonly studied in the Burdett-Mortensen literature and a valid approximation, under plausible calibrations. As an alternative, I could have incorporated a time dimension into the representative agent framework. Doing so would add complexity without altering the key mechanism underlying the result—that job acceptance decisions reflect worker expectations about future wage growth opportunities.

<sup>4</sup>These preferences are a close supply-side analogue to forms used to rationalize two aggregator demand systems for goods, following Fally (2022). Their homothetic restriction belongs to the class of Homothetic Indirectly-Implicitly Additive (HIIA) aggregators. First introduced by Hanoch (1975), the HIIA class is well-known in the literature studying monopolistic competition. See Matsuyama (2023) for a detailed review of HIIA and related forms.

condition aligns with Hornstein *et al.*'s (2011) observation that aggregate wage inequality in frictional models is constrained by the features of the search technology that workers employ. Crucially, it implies that wage inequality can be viewed as the outcome of a labor allocation problem faced by a representative agent whose preferences integrate income effects through the wage shifter  $K(\Lambda)$ .

Although such income effects are absent from the homothetic forms common in random utility models, my results reveal that search frictions do not necessarily force non-homothetic preferences. In particular, I show that homotheticity attains when workers' outside option scales proportionately with the average wage. In that case, the aggregator  $\Lambda$  is proportional to indirect utility, and the corresponding labor supply system satisfies Gorman-Pollak form (Gorman 1995, Pollak 1972), with CES nested as a special case.<sup>5</sup> Crucially, this form can also be derived from aggregating over rational workers with random utility.<sup>6</sup> Thus, the presence or absence of income effects does not mark a substantive difference between the approaches: one can adjust the representation of search-theoretic monopsony to map into a homothetic random utility model, and vice versa.

The existence of a representative agent formulation implies that standard tools in consumer theory (e.g., compensating variation) can be used to make welfare statements in job-ladder models.<sup>7</sup> As an illustration, I derive a parsimonious formula for calculating the welfare gains from employer entry—a “love-for-variety” effect that remains valid under both frameworks, in the spirit of Matsuyama & Ushchev (2023). This formula yields new theoretical and quantitative insights into the normative implications of job-ladder cyclicity (Moscarini & Postel-Vinay 2018), revealing that employer entry affects welfare through competing channels: a positive employment effect as more workers find jobs above reservation wage, and a negative reservation wage effect as job creation raises workers' outside options. Crucially, under a search interpretation, the strength of these two effects can be cast in terms of a few standard labor market statistics.

The unemployment rate plays a pivotal role within these statistics: when unemployment is high, there is substantial scope for welfare gains via increased employment, while the reservation wage is relatively unresponsive to job creation. As unemploy-

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<sup>5</sup>In particular,  $n_\omega = \ell_\omega \left( \frac{w_\omega}{V} \right) / A$ , for  $A \equiv \int_\Omega w_\omega \ell_\omega \left( \frac{w_\omega}{V} \right) d\omega / c$  and  $c > 0$  summarizing search parameters.

<sup>6</sup>I show this by adopting the arguments in Thisse & Ushchev (2016) and Trottner (2023).

<sup>7</sup>In particular, the unified framework can be used to assess how labor supply structure impacts aggregate efficiency under different demand-side formulations, thereby bridging existing work from the search-theoretic (e.g., Gautier *et al.* 2010) and monopsonistic competition literature (e.g., Trottner 2023).

ment falls, however, the gains from employment weaken while the reservation wage becomes increasingly sensitive to employer entry. These dynamics imply the existence of an unemployment threshold below which increasing employer variety reduces welfare. Nevertheless, a calibration of the formula to U.S. data (1999–2024) reveals that welfare gains from job creation remain positive throughout the sample period and vary counter-cyclically—from modest levels during tight labor markets to significant increases during economic downturns.

By embracing random search as a labor supply technology, I depart from the conventional approach in the literature building on Burdett (1978) and Burdett & Mortensen (1998)—which typically focuses on characterizing equilibrium wage distributions with search embedded within broader theories of matching, bargaining, or efficiency wages.<sup>8</sup> From this conceptual departure, I provide, to the best of my knowledge, the first representative agent formulation for search models. The preferences emerging from this representation incorporate two important insights from the job-ladder literature: Manning’s (2003) conceptualization of labor market power and Hornstein *et al.*’s (2011) demonstration that search frictions impose technological constraints on worker flows.

Furthermore, my results provide a theoretical foundation for adopting non-CES specifications when modeling monopsony arising from idiosyncratic worker tastes. The current literature typically relies on multinomial logit random utility specifications, which has made nested CES the dominant approach (e.g., Card *et al.* 2018, Lamadon *et al.* 2022, Berger *et al.* 2022). While nested CES offers many advantages, it also imposes limiting restrictions on, e.g., cross-sectional markdowns or wage-productivity pass-throughs.<sup>9</sup> I show that alternative functional forms, which naturally derive from either random utility or search behavior, can overcome these limitations while retaining tractability.

Beyond macro-labor applications, the representative agent formulation I develop offers a tractable building block for general equilibrium models, with potential appli-

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<sup>8</sup>In addition to specifying labor demand in detail, much of the BM literature has focused on extending the Burdett-Mortensen framework to allow for two-sided heterogeneity. See, e.g., Postel-Vinay & Robin (2002), Burdett & Coles (2003), Hagedorn & Manovskii (2013), Moscarini & Postel-Vinay (2013), Moscarini & Postel-Vinay (2016), Lise & Robin (2017), or Burdett *et al.* (2020).

<sup>9</sup>See Manning (2021), Card (2022), and Kline (2025a) for recent surveys of the reduced-form evidence on monopsonistic wage-setting and firm wage effects, and Kline (2025b) for an overview of this literature’s theoretical underpinnings. Using structural IO methods, Tortarolo & Zarate (2018), Dolfen (2020), and Yeh *et al.* (2022) document substantive cross-sectional markdown variation among firms in Colombia, Germany, and the US, respectively. For recent evidence on wage pass-through heterogeneity across firms see, e.g., Chan *et al.* (2023) and Garin & Silvério (2023).

cations in international trade (Helpman *et al.* 2010; MacKenzie 2018; Trottner 2020; Jha & Rodriguez-Lopez 2021; Felix 2022; Gutiérrez 2023), spatial economics (Lindenlaub *et al.* 2022; Bilal 2023; Heise & Porzio 2022; Kuhn *et al.* 2024), economic growth (Gouin-Bonenfant 2022; Garibaldi & Turri 2024), and other applied general equilibrium fields.

Finally, my results hold broader implications beyond labor markets. The monopolistic competition literature, for example, has developed various non-CES aggregators to analyze, e.g., variable markups (see Matsuyama (2023, 2025) for comprehensive surveys). The preferences over jobs representing the search technology can be mapped into the nested two-aggregator demand system studied by Arkolakis *et al.* (2019), Bertolotti & Etro (2022), and Fally (2022). By deriving this demand system from random search, I offer novel microfoundations for indirectly-implicitly additive utility structures.<sup>10</sup> The empirical tractability of measuring love-for-variety effects using readily observable statistics is merely one illustration of the potential value of this microfoundation.

The remainder of the paper proceeds as follows. Section 2 introduces a baseline representative agent framework for analyzing monopsony based on employer differentiation. Section 3 presents the canonical job ladder model, focusing on the optimal job search behavior of workers. Section 4 establishes the main theoretical result linking the two approaches. Section 5 studies the implications for the welfare gains from job creation. Section 6 concludes.

## 2 Representative agent framework

This section establishes a representative-agent framework for modeling labor market monopsony based on non-homothetic preferences over firm-level labor allocations. While it may seem abstract initially, Section 4 demonstrates that this preference structure emerges directly from optimal job choice under both search frictions and idiosyncratic worker-firm preferences.

### 2.1 Setup

Consider a static economy with a single primary factor, called labor, a representative household, and firms indexed by  $\omega \in \Omega$ , where  $\Omega$  is a Borel set with measure  $|\Omega|$ . The

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<sup>10</sup>Moreover, my results also clarify that two popular demand system classes — Homothetic with a Single Aggregator (Matsuyama & Ushchev 2017) and Homothetic Directly Implicitly Additive (Kimball 1995) — cannot arise from random search.

household supplies labor and demands an aggregate final good, whose price is chosen as the numeraire. Labor is traded in a single spot market, where firms post wages under full commitment.

## 2.2 Preferences

The representative household commands a unit mass of identical and indivisible labor units, called workers. Each worker can work at one firm but a fraction  $1 - e$  must engage in a non-market activity that generates  $\underline{w}$  units of final good consumption. Given a vector of wage offers  $\mathbf{w} = \{w_\omega : \omega \in \Omega\}$ , the household chooses consumption and the measure of workers  $n_\omega$  to allocate to each firm  $\omega \in \Omega$ . The household's preferences over consumption and labor allocations can be represented by an indirect utility function  $\mathcal{V}$  that satisfies three conditions. The first condition states that indirect utility can be written as

$$\mathcal{V} = Y(\mathbf{w}, \Lambda) \frac{L(\Lambda)}{K(\Lambda)}, \quad (1)$$

where  $Y(\mathbf{w}, \Lambda) \equiv \frac{1}{e} \int_{\Omega} n_\omega w_\omega d\omega$  denotes total employment income and  $\Lambda$  is a wage aggregator defined as  $\Lambda := \Lambda\left(\frac{\mathbf{w}}{\underline{w}}\right)$ . Here, the normalization of  $\mathbf{w}$  by  $\underline{w}$  incorporates how the relative value of each firm's wage offer varies depending on the non-market option. In turn,  $L(\Lambda)$  and  $K(\Lambda)$  are strictly positive and continuously differentiable functions. Here,  $L(\Lambda)$  captures the surplus from engaging in market employment, while  $K(\Lambda)$  acts as a wage shifter that adjusts the value of employment income for outside option.

The second condition imposes that  $K$  and  $Y$  jointly satisfy

$$\int_{\Omega} \int_{\underline{z}}^{\frac{w_\omega}{Y(\mathbf{w}, \Lambda)} K(\Lambda)} \ell_\omega(\xi) d\xi d\omega = 1, \quad (2)$$

where  $\underline{z} \geq 0$  is a constant and  $\ell_\omega$  is strictly positive for all  $\omega$ . This condition incorporates the requirement that total employment income is consistent with the distribution of worker surplus implied by the labor allocation across firms. The upper limit of the inner integral,  $w_\omega K/Y$ , implies that the relative value of a wage offer  $w_\omega$  depends on its comparison with other wage offers (through the aggregate  $Y$ ) and with the non-market return  $\underline{w}$  (via  $K$ ).

The third condition requires that the aggregator  $\Lambda$  satisfies the envelope condition

$$\frac{\partial \ln Y(\mathbf{w}, \Lambda)/K(\Lambda)}{\partial \ln \Lambda} + \frac{\partial \ln L(\Lambda)}{\partial \ln \Lambda} = 0. \quad (3)$$

This condition essentially balances the benefits of redistributing workers among firms with the cost in terms of lost surplus, ensuring the reservation wage adjusts appropriately. In other words, (3) guarantees that at the utility-maximizing labor allocation across firms, any shift in the adjusted total wage income  $Y/K$  induced by changes in  $\Lambda$  is exactly offset by its effect on the surplus from market employment, as captured by  $L$ . This balancing act is central to the determination of the reservation wage, defined as

$$w_r \equiv \underline{z} \frac{Y(\mathbf{w}, \Lambda)}{K(\Lambda)},$$

which emerges as the crucial link between the envelope condition and the resource constraint. The reservation wage adjusts to ensure that the labor allocation across firms is feasible under the resource constraint and consistent with utility maximization. Crucially, because  $w_r$  depends on the normalized vector  $\mathbf{w}/\underline{w}$  of wage offers, a shift in  $\mathbf{w}$  generally spells non-proportional adjustment of the reservation wage when  $\underline{w}$  is held fixed. As a result, a uniform scaling of wage offers does not translate into a proportional scaling of  $\mathcal{V}$ ; rather, the resulting shift in the reservation wage alters the household's marginal utility of income. As will become clear below, this structure spells non-homotheticity, whereby the relative value of different job offers varies with the level of wage offers.

Readers familiar with the literature on monopolistic competition may recognize that the formulation of indirect utility in (1)-(3) is reminiscent of the forms that Fally (2022) shows rationalizes separable demand systems with two aggregators. To ensure the regularity of  $\mathcal{V}$ , I impose

ASSUMPTION 1. Define  $m_\omega(z_\omega, \mathbf{z}) \equiv \frac{z_\omega \ell_\omega(z_\omega)}{\int_\Omega z_\omega \ell_\omega(z_\omega) d\omega}$  and  $\varepsilon_\omega(z) \equiv \frac{\partial \ln \ell_\omega(z)}{\partial \ln z}$ .

- a.  $\forall \omega \in \Omega, \Lambda > 0$ , and  $z > \underline{z}$ ,  $\varepsilon_{\ell_\omega}(z) > 0$  and  $\left( \frac{\partial \ln L/K}{\partial \ln \Lambda} + \varepsilon_\omega \frac{\partial \ln L}{\partial \ln \Lambda} \right) m_\omega(z) + \varepsilon_\omega(z) > -1$ .
- b.  $\forall \omega \in \Omega, \mathbf{w} > \mathbf{0}$ , and  $\underline{w} > 0$ ,  $\exists \Lambda$  s.t.  $\frac{\underline{w}}{Y(\mathbf{w}, \Lambda)} \ell_\omega \left( \frac{\underline{w}}{Y(\mathbf{w}, \Lambda)} K(\Lambda) \right) = 1/|\Omega|$ .

Under these regularity conditions,  $\mathcal{V}$  defines a well-behaved utility (continuous, increasing and quasi-convex in  $\mathbf{w}$  and  $\underline{w}$ ) and Roy's identity can be used to recover the labor supply system for firms  $\omega \in \Omega$ . See Appendix B for details.



### 2.3 Firm-level labor supply

Under the preference formulation described above, the labor supply  $n_\omega$  to a firm  $\omega \in \Omega$  is equal to

$$n_\omega = \ell_\omega \left( \frac{w_\omega}{Y(\mathbf{w}, \Lambda)} K(\Lambda) \right), \quad (4)$$

if  $w_\omega \geq \underline{z} \frac{Y}{K(\Lambda)}$ , and  $n_\omega = 0$ , otherwise. In addition, an adding-up constraint determines  $\Lambda$  (and  $Y$ ) via the requirement that the market shares of all employers sum to unity:

$$\int_{\Omega} \frac{w_\omega}{Y(\mathbf{w}, \Lambda)} \ell_\omega \left( \frac{w_\omega}{Y(\mathbf{w}, \Lambda)} K(\Lambda) \right) d\omega = 1. \quad (5)$$

Equation (4) implies that firms with relatively attractive wage offers secure a larger share of workers, where the attractiveness of a firm's wage offer depends on the normalized wage  $z_\omega \equiv \frac{w_\omega}{Y} K$ . The aggregator  $Y$  can be interpreted as an index of the fierceness of wage competition among employers, while the wage shifter  $K(\Lambda)$  determines how competitive pressures in the labor market vary with the relative value of the outside option. From Assumption 1 it intuitively follows that an increase in  $\underline{w}$  weakly reduces the labor supply to each firm

The normalized wage  $z_\omega$  also determines the wage elasticity of a firm's labor supply, given by

$$\varepsilon_\omega(z) \equiv \frac{\partial \ln n_\omega}{\partial \ln w_\omega} = \frac{\partial \ln \ell_\omega(z)}{\partial \ln z} > 0.$$

The wage elasticity function  $\varepsilon_\omega(\cdot)$  makes clear that the curvature of the function  $\ell_\omega$  plays a central role in determining the extent of employer market power. In particular, the shape of  $\ell_\omega$  captures the imperfections in employer substitutability that stem from the aggregate resource constraint imposed earlier.

Since Assumption 1 does not restrict the (upward-sloping) shape of  $\ell_\omega$ , the labor supply system for firms  $\omega \in \Omega$  in (4)-(5) is substantially more flexible than the nested CES forms typically deployed in the literature studying monopsony based on employer differentiation. To illuminate the connection to this literature, it is useful to consider the limiting case where  $K(\Lambda)$  and  $L(\Lambda)$  are constant.<sup>11</sup> In this case, indirect utility  $\mathcal{V}$  coincides with employment income  $Y$  (up to a linear transformation) and equation (2) simplifies to:

$$c = \int_{\Omega} \int_{\underline{z}}^{w_\omega/\mathcal{V}(\mathbf{w})} \ell_\omega(\xi) d\xi d\omega, \quad (6)$$

<sup>11</sup>This occurs, for instance, when  $\underline{w} = \gamma Y$  for some constant  $\gamma < 1$ .

for some constant  $c > 0$ . Under this homothetic restriction,  $\mathcal{V}$  hence belongs to the class of homothetic implicit indirectly additive (HIIA) aggregators, introduced by Hanoch (1975). Under HIIA, the labor supply for each firm  $\omega \in \Omega$  can be expressed as

$$\frac{n_\omega w_\omega}{Y} = \frac{\ell_\omega\left(\frac{w_\omega}{\mathcal{V}}\right)}{\int_\Omega w_\omega \ell_\omega\left(\frac{w_\omega}{\mathcal{V}}\right) d\omega}, \quad (7)$$

if  $w_\omega > \underline{z}\mathcal{V}$ , and  $n_\omega = 0$  otherwise.

Notice that equation (7) nests the canonical CES labor supply system as a special case, when  $\ell_\omega(z) = a_\omega z^{1+\beta}$ , for  $(\beta, \{a_\omega : \omega \in \Omega\}) > \mathbf{0}$ , and  $\underline{z} = 0$ . Compared to CES, HIIA permits two empirically important features of labor markets; namely, varying wage elasticities of labor supply across firms (which in turn generate endogenous wage markdowns in models of monopsonistic competition) and an endogenous reservation wage,  $w_r = \underline{z}\mathcal{V}$ , if  $\underline{z} > 0$ . In turn, the non-homothetic generalization of HIIA discussed in the previous section allows for the possibility that neither the reservation wage nor the index of competitive labor market pressures scale linearly with wage offers. Both of these possibilities generically arise in the canonical search-theoretic monopsony model, which I lay out in the next section.

### 3 Search framework

This section outlines the canonical search-theoretic approach to modeling monopsony, based on Burdett (1978) and Burdett & Mortensen (1998) (henceforth, BM). In this model, time flows continuously, and the labor market exhibits search frictions. A unit mass of risk-neutral workers divides between employed ( $e$ ) and non-employed ( $u = 1 - e$ ) states. Potential employers are indexed by  $\omega \in \Omega$ , where  $\Omega$  is a Borel set of measure  $|\Omega|$ . Each firm  $\omega$  posts a wage  $w_\omega$  under full commitment. Search is random, and characterized by a technology  $\phi$ .

#### 3.1 Search

A random search technology comprises the following elements: a discount rate  $r$ , a vector  $\{\lambda_\omega : \omega \in \Omega\}$  of Poisson rates governing the frequency at which non-employed workers make contact with each firm, a parameter  $\chi \in (0, 1]$  capturing the relative efficiency of on-the-job versus off-the-job search, a job destruction rate  $\sigma$ , and the flow value  $\underline{w}$  of non-employment. Bundling these parameters, the vector  $\phi \in \Phi$  defines a

random search technology.

DEFINITION 1.  $\Phi := \{ \phi = (r, \{ \lambda_\omega : \omega \in \Omega \}, \chi, \sigma, \underline{w}) : r, \sigma > 0, \chi \in (0, 1], \int_\Omega \lambda_\omega d\omega < \infty \}$

Aggregating firm-level contact rates yields a total employer contact rate of  $\lambda \equiv \int_\Omega \lambda_\omega d\omega$  for non-employed workers, and  $\lambda\chi$  for employed workers. It is assumed that workers move jobs instantaneously whenever they accept an offer; and with strictly positive probability whenever indifferent between an offer and their current state.<sup>12</sup>

In this environment, the optimal job search behavior of workers gives rise to state-dependent value functions. For non-employed workers, the value function  $U$  captures both the immediate flow value of non-employment and the option value of future employment

$$rU = \underline{w} + \lambda \int_\Omega [V(w_\omega) - V(w_r)] dF(w_\omega | w_r),$$

For employed workers, the value function  $V(w)$  incorporates their current wage, the risk of job loss, and the prospect of finding better-paid employment:

$$rV(w) = w + \sigma [U - V(w)] + \lambda\chi \int_\Omega [V(w_\omega) - V(w)] dF(w_\omega | w).$$

Here, the function  $F(w' | w)$  represents the distribution of wages among job offers that a worker currently earning  $w$  would accept,

$$F(w' | w) = |\Omega|^{-1} \int_\Omega \frac{\lambda_\omega}{\lambda} \mathbf{1}\{w \leq w_\omega \leq w'\} d\omega, \quad (8)$$

and is assumed to be continuously differentiable and non-atomic on the interior of its domain.<sup>13</sup>

The above Bellman equations incorporate optimal job search behavior: Employed workers optimally transition to a new job whenever they receive a higher wage offer, while non-employed workers decide on a reservation wage  $w_r$ , to satisfy the indifference condition  $U = V(w_r)$ . Standard arguments, which can be found in Appendix A.2,

<sup>12</sup>This assumption rules out labor allocations in which workers never move when  $w_\omega = w \forall \omega \in \Omega$ . While the BM literature uses this assumption to eliminate symmetric wage equilibria (see, e.g., Shimer 2006); here, its purpose is to facilitate the interpretation of the welfare measure introduced, later, in Section 5.

<sup>13</sup>Here, the indicator function  $\mathbf{1}\{w' \leq w_\omega \leq w\}$  selects the firms  $\omega \in \Omega$  offering wages in the interval  $[w', w]$ . The factor  $\lambda_\omega/\lambda$  ensures that the distribution is properly normalized to account for heterogeneity in contact rates. In writing the value functions, I implicitly assumed  $w_\omega := w(\omega)$  to be a continuously differentiable mapping in  $\omega$ , so that the Jacobian  $dw_\omega = \frac{\lambda_\omega}{\lambda} d\omega$  is well-defined. See Appendix A.1. for details.

yield the reservation wage as the sum of the non-employment benefit plus the expected gain from waiting for a better job

$$w_r = \underline{w} + \lambda(1 - \chi) \int_{w_r}^{\infty} \frac{1 - F(w|w_r)}{r + \sigma + \lambda\chi[1 - F(w|w_r)]} dw. \quad (9)$$

### 3.2 Firm-level labor supply

The steady-state distribution of employment arises from balanced flows between employment states and across the wage distribution. Without loss of generality, suppose that  $w_\omega \geq w_r$  for all  $\omega \in \Omega$ . Then, the non-employment and employment rates solve the following steady state conditions:

$$u = \frac{\sigma}{\sigma + \lambda}, \quad e = 1 - u = \frac{\lambda}{\sigma + \lambda}. \quad (10)$$

For wages above the reservation level, the outflow rate to non-employment and employment at higher wages is given by  $\sigma + \lambda\chi [1 - F(w|w_r)]$ , while the inflow rate from non-employment is  $\lambda F(w|w_r)$ . Thus, the cumulative fraction of workers employed at wage below  $w$  is given by

$$G(w) := G(w, w_r; \phi) = \frac{F(w|w_r)}{1 + \frac{\lambda\chi}{\sigma} [1 - F(w|w_r)]}, \quad (11)$$

if  $w \geq w_r$ , and  $G(w) = 0$ , otherwise.

For firm  $\omega$ , total worker inflows consist of hires from non-employment and transitions from lower-wage firms,  $\lambda_\omega [u + e\chi G(w_\omega)]$ . The corresponding outflow rate to non-employment or higher-wage firms is  $\sigma + \lambda\chi [1 - F(w_\omega|w_r)]$ . In steady state, these flows balance, determining firm  $\omega$ 's employment level:

$$n_\omega(w_\omega, \phi) = \frac{\lambda_\omega}{\sigma + \lambda} \frac{1 + \frac{\lambda\chi}{\sigma}}{\left[1 + \frac{\lambda\chi}{\sigma} [1 - F(w_\omega|w_r)]\right]^2}, \quad (12)$$

Equations (9) and (12) characterize the labor supply system generated by the search technology  $\phi$  for a given a vector of wage offers  $\mathbf{w}$  and non-employment flow value  $\underline{w}$ . The corresponding total utilitarian welfare flow can be obtained from summing across

the values of all workers, both non-employed and employed:<sup>14</sup>

$$V = ruV(w_r; \phi) + r \int_{\Omega} V(w_{\omega}, \phi) n(w_{\omega}; \phi) d\omega. \quad (13)$$

The steady-state labor supply system exhibits structural similarities to the preference-based framework developed in Section 2. In both cases, firms face upward-sloping labor supply curves, with elasticities that vary systematically with their position in the wage offer distribution. As I elaborate in the next section, this similarity in fact reflects a positive and normative equivalence.

## 4 Equivalence Result

This section establishes the formal equivalence between the search-based labor supply model in Section 3 and the representative agent framework outlined in Section 2. The core insight is that the job-ladder model’s labor allocation can be derived from the maximization of a non-homothetic preference over firm-level labor allocations. The analysis proceeds by constructing an indirect utility function that reproduces the steady-state employment distribution derived from search behavior.

### 4.1 Preliminary Lemmas

This section lays out three lemmas that progressively build toward the representation theorem. First, I characterize how welfare depends on reservation wages and mean wages; second, I establish how reservation wages balance employment and non-employment flows; and third, I show that the welfare function satisfies an analogue of Roy’s identity. Together, these lemmas demonstrate that optimal search behavior generates exactly the non-homothetic labor supply system described in Section 2.

Following standard practice, I focus on a setting that the BM literature often refers to as a “timeless” equilibrium, where  $r/\lambda \rightarrow 0$ .<sup>15</sup> Under this limit, time discounting becomes negligible, and every worker’s steady-state value converges to the same common level. The lemma below characterizes this welfare level.

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<sup>14</sup>As will become clear in the next section, values are scaled to ensure that utilitarian welfare remains finite in the limiting case  $r/\lambda \rightarrow 0$ .

<sup>15</sup>This approximation is also valid since  $r$  is negligible compared to  $\lambda$  in plausible calibrations of the model. I could have, alternatively, incorporated a time dimension into the representative agent framework. Doing so would add complexity without fundamentally altering the main result.

LEMMA 1. For any search technology  $\phi \in \Phi$ , vector  $\mathbf{w}$  of wage offers and non-employment income  $\underline{w}$ ,

$$\mathcal{V}(\mathbf{w}, \underline{w}) \equiv \lim_{r/\lambda \rightarrow 0} rV(w_\omega) = \frac{\sigma}{\sigma + \lambda\chi} w_r(\mathbf{w}, \underline{w}) + \frac{\lambda}{\sigma + \lambda\chi} W(\mathbf{w}, \underline{w}), \quad (14)$$

where the reservation wage  $w_r$  and the mean wage  $W$  satisfy, respectively,

$$w_r(\mathbf{w}, \underline{w}) \equiv \chi \underline{w} + (1 - \chi) \mathcal{V}, \quad (15)$$

$$W(\mathbf{w}; \underline{w}) \equiv \left(1 + \frac{\sigma}{\lambda}\right) \int w_\omega n_\omega(w_\omega, w_r) d\omega, \quad (16)$$

with  $n_\omega(w_\omega, w_r)$  given in (12).

*Proof.* See Appendix A.3. □

Equation (14) shows that welfare is a weighted sum of the reservation wage  $w_r$  – which marks the entry point into the labor market – and the mean wage  $W$ , representing a worker’s expected earnings from climbing the job ladder.<sup>16</sup> The key economic parameters governing this weighted average are  $\sigma$  (the rate at which workers exit employment),  $\lambda$  (the rate at which job offers arrive), and  $\chi$  (the relative efficiency of on-the-job search). Higher values of  $\lambda/\sigma$  and  $\chi$  increase the relative importance of the mean wage, as workers spend more time employed and move up the job ladder more quickly.

Equation (15) illuminates how  $\chi$  shapes the reservation wage  $w_r$ . Intuitively, when job offers arrive faster during employment ( $\chi \uparrow$ ), workers lower their reservation wages because being employed now means sacrificing fewer future job opportunities. As  $\chi \rightarrow 0$ , so that on-the-job search becomes impossible, then  $w_r \rightarrow \mathcal{V}$ , meaning no worker would accept a job paying below their lifetime expected income.<sup>17</sup> Conversely, when  $\chi \rightarrow 1$  and employment does not impede future search prospects,  $w_r \rightarrow \underline{w}$ , causing workers to accept any wage offer above their outside option. Between these extremes,  $w_r$  balances the value of market versus non-market time in a way that supports the steady-state labor allocation.

LEMMA 2. Under the definitions in Lemma 1,  $w_r(\mathbf{w}, \underline{w})$  in (15) solves

<sup>16</sup>One can re-arrange equations (14) and (15) to see that  $\mathcal{V}$  also coincides with expected income,  $\mathcal{V}(\mathbf{w}, W) = \frac{\sigma}{\sigma + \lambda} \underline{w} + \frac{\lambda}{\sigma + \lambda} W$ , when  $r/\lambda \rightarrow 0$ . This characterization is known in the BM literature (see, e.g., Bilal & Lhuillier 2022) but does not explain how the reservation wage links  $\mathcal{V}$ ,  $W$  and  $\underline{w}$ .

<sup>17</sup>This limiting case corresponds to Diamond’s 1971 paradox, yielding zero worker surplus in any labor market equilibrium.

$$\int_{\Omega} \left[ \int_{1+\frac{\sigma}{\lambda}}^{(1+\frac{\sigma}{\lambda})\frac{w\omega}{w_r(w,w)}} \ell_{\omega}(\xi) d\xi \right] d\omega = \frac{\sigma}{\sigma + \lambda\chi}, \quad (17)$$

where

$$\ell_{\omega} \left( \frac{w}{A} \right) = \frac{\lambda\omega}{\sigma + \lambda} \frac{1 + \frac{\lambda\chi}{\sigma}}{\left( 1 + \frac{\lambda\chi}{\sigma} [1 - F \left( \frac{w}{A} \right)] \right)^2}, \quad \text{for } \omega \in \Omega, A > 0 \quad (18)$$

and

$$F \left( \frac{w}{A} \right) \equiv \frac{1}{|\Omega|} \int_{\Omega} \frac{\lambda\omega}{\lambda} \mathbf{1} \left\{ \frac{\sigma + \lambda}{\sigma} \leq \frac{w\omega'}{A} \leq \frac{w}{A} \right\} d\omega'. \quad (19)$$

*Proof.* See Appendix A.4. □

The right-hand side of equation (17) is the fraction of time that a worker spends in non-employment in the steady state. Lemma 2 says that this fraction must match the aggregate of firm-level worker surpluses on the left-hand side, ensuring equilibrium flow balance. If workers raise the reservation wage  $w_r$ , fewer accept low-paying jobs and more remain out of work, so non-employment time increases; but simultaneously, those who do accept end up with higher wages, which bolsters the total surplus from employment. The equality in Lemma 2 enforces that these opposing effects cancel precisely at the equilibrium reservation wage.

In Lemma 2, each function  $\ell_{\omega}$  evaluates the marginal surplus a firm generates if its posted wage  $w$  exceeds a common reference wage  $A$  by a factor  $\xi = w/A$ . Summing  $\ell_{\omega}(\xi)$  over  $[1/e, w_{\omega}/A]$  for each firm  $\omega$  and integrating over all firms yields the average worker surplus on the left-hand-side. Note that  $\ell_{\omega}$  coincides with the actual labor supply in (12) if and only if the reference wage  $A$  is precisely equal to the reservation wage in (15) scaled by the steady state measure of employed workers. Under that condition, the optimal job search behavior of workers is in line with the time workers spend on each rung  $\omega$  of the job ladder.

In effect, Lemma 2 yields an aggregate envelope condition: it shows that in a labor supply equilibrium, any marginal increase in the value of non-employment is precisely counterbalanced by adjustments in the total surplus that employed workers earn by moving up the job ladder. This balancing act has a powerful implication for how the reservation wage  $w_r$  co-moves with non-wage income  $\underline{w}$  and the mean wage. From equations (14)-(15), this co-movement can be described by

$$\frac{w_r}{\underline{w}} = 1 + (1 - \chi) \frac{\lambda}{\sigma + \lambda} \left( \frac{W}{\underline{w}} - 1 \right)$$

Intuitively, when  $\underline{w}$  rises, then the reservation wage  $w_r$  and the mean wage  $W$  must shift so that the share of workers staying out of low-paid employment remains consistent with flow-balance constraints. This observation prepares the ground for showing how the function  $\mathcal{V}(\mathbf{w}, \underline{w})$  can be described by a representative-agent viewpoint: once  $w_r$  must respond to changes in  $\underline{w}$  according to Lemma 2, it acts like a common wage shifter in the labor supply system for firms. adjusting to ensure that allocations are consistent with aggregate resource constraints. Lemma 3 makes this link explicit by showing that  $\mathcal{V}$  satisfies an analog of Roy's identity in  $\left(\frac{\mathbf{w}}{eW}, \frac{W}{\underline{w}}\right)$ -space.

LEMMA 3. *Under the definitions in Lemma 1 and Lemma 2, there exists a wage index  $\Lambda(W, \underline{w}) = W \left(\frac{\mathbf{w}}{\underline{w}}, 1\right)$  and a function  $K : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  so that*

$$\frac{\frac{\partial \ln \mathcal{V}}{\partial \ln w_\omega}}{\frac{\partial \ln \mathcal{V}}{\partial \ln \underline{w} e \Lambda}} = \frac{w_\omega \ell_\omega \left(\frac{w_\omega}{eW} K(\Lambda)\right)}{\int_{\Omega} w_\omega \ell_\omega \left(\frac{w_\omega}{eW} K(\Lambda)\right) d\omega'}, \quad \forall \omega \in \Omega, \quad (20)$$

where  $\int_{\Omega} \frac{\frac{\partial \ln \mathcal{V}}{\partial \ln w_\omega}}{\frac{\partial \ln \mathcal{V}}{\partial \ln \underline{w} e \Lambda}} d\omega = 1$ .

*Proof.* See Appendix A.5. □

The key step in proving Lemma 3 is to establish that the welfare function  $\mathcal{V}$  satisfies an envelope condition with respect to the employment wage premium  $W/\underline{w}$ . In doing so, Lemma 3 bridges the search-based approach and the representative-agent viewpoint: it show that changes in  $w_\omega$  and  $\underline{w}$  (when appropriately scaled) feed into  $\mathcal{V}$  as they would exactly in a standard indirect utility framework with Roy's identity.

## 4.2 Representation Theorem

Armed with Lemmas 1 to 3, I am now ready to state the main result, which shows that the welfare function  $\mathcal{V}$  can be written as an indirect utility function in the form described in section 2.

THEOREM 1. *Under the definitions in Lemmas 1-3,  $\mathcal{V}$  is formally equivalent to an indirect utility function of the form*

$$\mathcal{V} = eW \frac{L\left(\frac{W}{\underline{w}}\right)}{K\left(\frac{W}{\underline{w}}\right)}, \quad (21)$$



where

$$\frac{\sigma}{\sigma + \lambda\chi} = \int_{\Omega} \int_{\frac{\frac{w_{\omega}}{\lambda} W}{\lambda + \sigma}}^{\frac{w_{\omega}}{\lambda} K(\frac{W}{w})} \ell_{\omega}(\xi) d\xi d\omega, \quad \frac{\partial \mathcal{V}(\mathbf{w}, \frac{W}{w})}{\partial \frac{W}{w}} = 0, \quad (22)$$

and

$$K(\Lambda) \equiv \frac{\Lambda}{1 + (1 - \chi)\frac{\lambda}{\sigma + \lambda}(\Lambda - 1)}, \quad L(\Lambda) \equiv \frac{\sigma + \lambda}{\lambda\chi + \sigma} \frac{\sigma}{\lambda} + \frac{\sigma + \lambda}{\lambda\chi + \sigma} \chi K(\Lambda). \quad (23)$$

*Proof.* See Appendix A.6. □

Theorem 1 provides a fundamental link between search models and representative agent frameworks: it rationalizes the labor supply system emerging from BM as the choice of a representative agent with non-homothetic preferences over firm-level labor allocations. This equivalence implies that search frictions can manifest in wage disparities that are mathematically equivalent to variations in preferences across similar workers—both mechanisms ultimately lead to the same allocation of workers across firms with varying wage offers. Corollary 1 attains the labor supply system generated by  $\mathcal{V}$ .

**COROLLARY 1.** *Under the definitions in Lemma 2 and Theorem 1, the labor supply system for  $\omega \in \Omega$  generated by (21)-(24) can be written*

$$n_{\omega} = \ell_{\omega} \left( \frac{w_{\omega}}{eW} K \left( \frac{W}{w} \right) \right), \quad (24)$$

where  $W$  satisfies the adding-up constraint:

$$\int_{\Omega} \frac{w_{\omega}}{eW} \ell_{\omega} \left( \frac{w_{\omega}}{eW} K \left( \frac{W}{w} \right) \right) d\omega = 1. \quad (25)$$

*Proof.* See Appendix A.6. □

Theorem 1 and Corollary 1 establish the existence of a representative-agent formulation for the search-theoretic model of monopsony described in Section 3. Given a vector of wage offers  $\mathbf{w}$  and the value of non-employment  $w$ , solving the fixed point equations (24)-(25) yields the equilibrium distribution of workers across firms, the mean wage  $W$ , the employment wage premium  $W/w$ , and the mean-min wage ratio  $K(\frac{W}{w})$ . Using these outcomes, utilitarian welfare flow  $\mathcal{V}$  can then be readily computed from equation (21). Appendix C provides a quantitative illustration.

Crucially, the above results hold regardless of how wages are determined on the labor-demand side—be it monopsonistic competition with exogenous contact rates

(amenity shifters) or search-and-matching with bargaining or efficiency wages. Below, I leverage Theorem 1 to establish the connection between job ladder and random utility models of labor market monopsony.

### 4.3 Relation to Random Utility Models

Call a search technology  $\phi$  homothetic if it implies a constant employment wage premium.

**DEFINITION 2.** *If  $\phi \in \Phi$  is homothetic, then there exists a  $\gamma < 1$  such that  $\underline{w} = \gamma W(\mathbf{w})$  for all  $\mathbf{w}$ .*

Definition 2 encompasses models where the value  $\underline{w}$  of non-employment is null; or, where  $\underline{w}$  captures an endogenous unemployment benefit funded by linear taxation of wage income. Moreover, it also accommodates models where non-employment poses a psychological cost in proportion to the mean wage of employed workers, when  $\gamma < 0$ .

Using Theorem 1, it is easy to show that homothetic search technologies can be represented by homothetic indirect utility functions.

**COROLLARY 2.** *Consider a homothetic search technology  $\phi \in \Phi$  with  $\underline{w} = \gamma W(\mathbf{w}, \phi)$  and  $\gamma < 1$ . Then,  $\mathcal{V} := \mathcal{V}(\mathbf{w})$  is formally equivalent to the indirect utility of a representative agent with homothetic preferences over firm-level labor allocations given by*

$$\int_{\Omega} \int_{\underline{z}}^{w_{\omega}/\mathcal{V}(\mathbf{w})} \ell_{\omega}(\xi; \phi) d\xi d\omega = \underline{z},$$

where

$$\ell_{\omega}(z_{\omega}; \phi) = \frac{\lambda_{\omega}}{\lambda} \left[ \frac{1 + \frac{\lambda \chi}{\sigma}}{1 + \frac{\lambda \chi}{\sigma} [1 - F(\xi | \underline{z})]} \right]^2,$$

$$\underline{z} := \underline{z}(\phi) \equiv \frac{\gamma + \frac{\lambda}{\sigma} [1 - \chi(1 - \gamma)]}{\gamma + \frac{\lambda}{\sigma}} \equiv \frac{w_r}{\mathcal{V}},$$

and

$$F(z | \underline{z}) \equiv \frac{1}{|\Omega|} \int_{\Omega} \frac{\lambda_{\omega}}{\lambda} 1 \left\{ \underline{z} \leq \frac{w_{\omega}}{\mathcal{V}} \leq z \right\} d\omega.$$

*Proof.* See Appendix A.7. □

From Corollary 2 it follows that, for homothetic search technologies, the indirect utility  $\mathcal{V}$  is equal to the dual wage index that represents the preference of labor allocations (up to a strictly increasing transformation). Following the earlier discussion in

Section 2, the labor supply system generated by this wage index can be written as

$$n_\omega \left( \frac{w_\omega}{\mathcal{V}} \right) = \frac{\ell_\omega \left( \frac{w_\omega}{\mathcal{V}} \right)}{\int_\Omega \frac{w_{\omega'}}{\mathcal{Y}} \ell_{\omega'} \left( \frac{w_{\omega'}}{\mathcal{V}} \right) d\omega'},$$

for  $\omega \in \Omega$ .

Now, consider a discrete choice model where the indirect utility  $u_i(\omega)$  of worker  $i$  when working for a firm  $\omega$  with wage offer  $w_\omega$  is given by

$$u_i(\omega; \phi) = \mu \ln w_\omega \ell_\omega \left( \frac{w_\omega}{\mathcal{V}} \right) + \varepsilon_i(\omega), \quad (26)$$

with the shocks  $\varepsilon_i(\omega)$  being i.i.d. Gumbel (with zero location and scale parameter  $\mu$ ). The supply system that arises from utility-maximizing job choices of a continuum of such workers coincides with the market shares representing the HIIA utility in Corollary 2.

**PROPOSITION 1.** *The multinomial logit model  $\mathbb{P}(\omega; \phi) \equiv \mathbb{P}[\omega = \arg \max_{\omega' \in \Omega} u_i(\omega'; \phi)]$  is formally equivalent to the labor supply system generated by the homothetic search technology  $\phi$  in Corollary 2.*

*Proof.* See Appendix A.8. □

The key insight of Proposition 1 is that a homothetic search technology generates the same employer choice probabilities as a logit discrete choice framework. This result makes precise how search frictions can, in the aggregate, yield equivalent limitations to worker mobility as taste idiosyncrasies. By establishing this equivalence, both Corollary 2 and Proposition 1 blur the traditional dichotomy between "frictional" and "taste-driven" labor market models, suggesting that what might seem like fundamentally different modeling approaches are, in fact, alternative representations of the same underlying economic mechanisms.

However, as the next section will demonstrate, while the positive predictions of these models coincide, their welfare and policy implications can differ substantially depending on how the parameters characterizing the indirect utility respond to policy. Under search-based microfoundations, policies that enhance job search efficiency naturally alter worker-firm contact rates ( $\lambda$ ) and thereby transform the indirect utility function itself, whereas under pure preference specifications, these structural parameters are typically held fixed when evaluating policy counterfactuals.

## 5 Application

The isomorphism between models of preference-based job differentiation and the class of search models described in Section 3 implies that standard tools from consumer theory can be used to assess the structure of the underlying search technology shapes welfare and policy analysis in the BM environment. This section applies this insight to derive a simple formula for measuring how the gains from employer entry depend on the underlying search technology.

### 5.1 Measuring love-for-variety in random search models

Love-for-variety - productivity or utility gains from from increasing variety of differentiated goods - plays a central role in many fields of economics (Matsuyama 1995), including in international trade (Helpman & Krugman 1987; Melitz 2003), economic growth (Grossman & Helpman 1993; Gancia & Zilibotti 2005), and spatial economics (Fujita *et al.* 1999). In labor economics, however, love-variety has received relatively little attention, despite its natural connection to the question of how much workers benefit from having access to a larger variety of employers—in other words, the welfare gains from job creation.

In recent work, Matsuyama & Ushchev (2023) propose a measure of love-for-variety for general homothetic demand systems that can be defined solely in terms of the mass of available goods. Here, I leverage the results in section 4 to adapt this measure to study the gains from employer entry in the class of random search models discussed in Section 3.

For ease of exposition, I restrict attention to search technologies with a constant employment wage premium  $\gamma$  and symmetric contact rates  $\lambda_\omega \equiv \lambda_0$  for all firms. Call these technologies homothetic and symmetric. Denote  $\bar{\Omega}$  the set of all potential employers, and let the labor allocation vector  $\mathbf{n} = \{n_\omega : \omega \in \bar{\Omega}\}$  be defined over a set of available firms  $\Omega \subset \bar{\Omega}$  and the set of unavailable firms,  $\bar{\Omega} \setminus \Omega$ . That is  $n_\omega = 0$  for  $\omega \in \bar{\Omega} \setminus \Omega$ .

My goal is to study the effect changing the mass  $|\Omega|$  of available employers on welfare. To this end, denote symmetric wage patterns among all available firms by  $\mathbf{w} = w\mathbf{1}_\Omega$ , where  $w > 0$  is a scalar and the quantity vector  $\mathbf{1}_\Omega \equiv (\mathbf{1}_\Omega; \omega \in \bar{\Omega})$  is defined as follows:

$$(\mathbf{1}_\Omega)_\omega = \begin{cases} 1 & \text{for } \omega \in \Omega \\ 0 & \text{for } \omega \in \bar{\Omega} \setminus \Omega \end{cases},$$

which satisfies  $\int_{\Omega} (\mathbf{1}_{\Omega})_{\omega} d\omega = |\Omega|$ .

Under symmetric wage patterns and contact rates, Corollary 2 implies that welfare be written as  $\mathcal{V}(\mathbf{w}; \phi) = w\mathcal{V}(\mathbf{1}_{\Omega}, \phi)$ . Here, the function  $\mathcal{V}(\mathbf{1}_{\Omega}; \phi)$  captures the surplus workers earn due to employer variety. The measure of love-for-variety is then intuitively defined as the rate at which  $\mathcal{V}(\mathbf{1}_{\Omega}; \phi)$  increases when the mass  $|\Omega|$  of available employers rises. In other words, it corresponds to the welfare gain from a proportional increase in the measure of available employers while holding the wage of each firm constant.

**DEFINITION 3.** *For the subset of search technologies  $\phi \in \Phi$  that are symmetric and homothetic, define the love-for-variety measure as*

$$\mathcal{L}(|\Omega|; \phi) = \frac{d \ln \mathcal{V}(\mathbf{1}_{\Omega}; \phi)}{d \ln |\Omega|}.$$

This measure,  $\mathcal{L}$ , isolates the pure "supply-side" effects of employer variety on aggregate welfare, separate from other market factors. In other words, the effects measured by  $\mathcal{L}$  are relevant regardless of what is assumed on the demand side, that is the process generating the wage offers and contact rates of firms, which could be modeled as monopsonistic or oligopsonistic competition, with or without endogenous firm-worker contact rates and with or without firm heterogeneity. Moreover, Definition 3 does not presuppose or require the existence of a symmetric wage equilibrium.

## 5.2 Formula

I now derive the measure of love-for-variety for homothetic and symmetric search technologies. Using Corollary 2, it is easy to show that the function governing the gains from job variety is given by

$$\mathcal{V}(\mathbf{1}_{\Omega}; \phi) = \frac{1}{\underline{z}(\mathbf{1}_{\Omega}; \phi)} \frac{\left(1 + |\Omega| \frac{\lambda_0 \chi}{\sigma}\right)^2}{\left(1 + |\Omega| \frac{\lambda_0 \chi}{\sigma}\right)^2 + 1}, \quad (27)$$

where

$$\underline{z}(\mathbf{1}_{\Omega}; \phi) \equiv \frac{\gamma + |\Omega| \frac{\lambda_0}{\sigma} [1 - \chi(1 - \gamma)]}{\gamma + |\Omega| \frac{\lambda_0}{\sigma}} \in (0, 1). \quad (28)$$

Differentiation of equations (27)-(28) then yields  $\mathcal{L}$ .

**PROPOSITION 2.** *For a homothetic and symmetric search technology  $\phi$ , the measure of love-for-variety can be written  $\mathcal{L}(M; \phi) := \mathcal{L}(u, \chi, \gamma)$ , and satisfies*

$$\mathcal{L} = \frac{2\chi(1-u)}{[u + \chi(1-u)] \left[1 + (1 + \chi(1-u)/u)^2\right]} - \frac{\chi\gamma(1-\gamma)(1-u)}{[\gamma + (1-\gamma)u] [\gamma + (1-\chi)(1-\gamma)u]}. \quad (29)$$

*Proof.* See Appendix A.9. □

Proposition 2 implies that  $\mathcal{L}$  can be written as a function of three parameters: the non-employment rate  $u$ , the relative search efficiency  $\chi \in (0, 1)$ , and the inverse employment wage premium  $\gamma$ . Below, I leverage this fact to quantify and decompose the gains from job creation in the US. Before doing so, I briefly pause to discuss a couple of noteworthy observations about Proposition 2.

First, Proposition 2 shows that employer entry impacts welfare through both an employment and a reservation wage effect. The employment effect, captured by the first term on the right-hand side of (29), captures the response of welfare when the reservation wage is held fixed. Under a fixed reservation wage, job creation raises welfare by implying that more workers now earn the surplus  $w - w_r$ . In turn, the reservation wage effect captures the intensive margin response of aggregate worker surplus. Since the reservation wage must rise to compensate workers for the increased opportunity cost of non-employment, job creation lowers aggregate welfare through this channel. Thus, the net welfare impact of employer entry may be positive or negative, depending on the relative strength of these two effects.

The tractability of deriving this clean expression for  $\mathcal{L}$  in Proposition 2 demonstrates the power of the representation theorem established in Section 4. Without the formal equivalence between search models and representative agent frameworks that I proved earlier, I would lack the theoretical tools to analyze love-for-variety in labor markets with such clarity. The mathematical structure uncovered in the isomorphism result serves as the foundation for translating complex labor market dynamics into a simple, measurable statistic that captures the welfare implications of changing employer variety.

Proposition 2 also delineates how the gains from job creation depend on the underlying search technology. Higher relative search efficiency  $\chi$  enhances the matching process between workers and jobs, thereby strengthening the positive extensive margin effect of job creation. At the same time, a higher  $\chi$  also makes workers' reservation wages more sensitive to improved employment prospects, strengthening the negative

intensive margin effect. The relative value of non-employment  $\gamma$  only impacts the intensive margin effect but has similarly non-linear effects on  $\mathcal{L}$  as  $\chi$ .

The non-employment rate  $u$  incorporates how  $\mathcal{L}$  depends on labor market tightness. Intuitively, when  $u$  is low, there is limited scope for firm entry to raise welfare via the employment effect. However, low  $u$  simultaneously raises the sensitivity of the reservation wage to employer entry, thereby amplifying the negative intensive margin effect. This intuitively suggests that job creation may cease to yield positive welfare benefits below a sufficiently low unemployment threshold. The Proposition below formally establishes the existence of such a threshold and characterizes its properties.

**PROPOSITION 3.** *For all  $\chi \in (0, 1)$  and  $\gamma \in (0, 1)$ , there exists a  $u^*(\chi, \gamma) \in (0, 1)$  such that  $\mathcal{L}(u, \chi, \gamma) > 0$  if, and only if,  $u > u^*(\chi, \gamma)$ . Moreover,  $\partial u^*/\partial \chi > 0$  and  $\partial u^*/\partial \gamma < 0$ .*

*Proof.* See Appendix A.10. □

For  $u$  above  $u^*$ , the benefits from greater employment outweigh the costs of upward reservation wage adjustment. However, once  $u$  is pushed below  $u^*$ , the negative intensive margin effect offsets the positive extensive margin effect. The fact that  $u^*$  rises with  $\chi$  indicates that when relative search efficiency is high, there is little additional boost to employment incentives at low levels of unemployment, while the resulting increase in reservation wage pressures weakens the net benefits from job variety. Conversely,  $u^*$  decreases in  $\gamma$  because a lower employment wage premium lowers the returns to employment, creating greater reservation wage pressures at any level of market tightness.

The above discussion makes clear that the gains from variety implied by the underlying search technology can differ both quantitatively and qualitatively depending on the state of the economy, the policy environment, or the relative efficiency of on-the-job search. I now turn to assessing the quantitative implications for the US labor market.

### 5.3 Quantitative implications

I use the measure of love-for-variety derived from the equivalence result in Section 4 to quantify the welfare gains from job creation implied by the class of search models defined in Section 3.

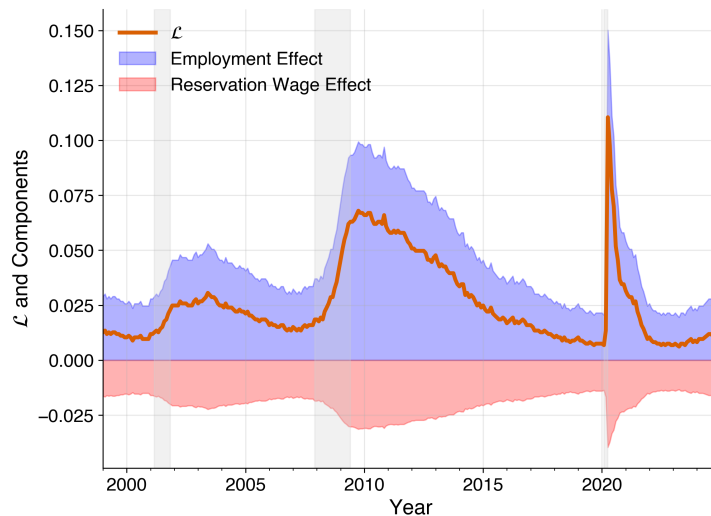


Figure 1: Estimates of  $\mathcal{L}(M)$  for the US Labor Market

*Note:* Baseline estimates of the welfare gains from employer entry in the US labor market implied by homothetic search technologies, based on evaluating equation (29) using monthly employment data from FRED, and setting  $\chi = 0.3$  and  $\gamma = 0.4$ . Decomposition into an extensive (blue) and intensive (red) margin effect based on the first and second summand on the right-hand-side of equation (29), respectively.

**Baseline calibration** As a baseline, I calibrate  $\mathcal{L}$  for the US labor market at a monthly frequency for the period 1999 to 2024. Data on the unemployment rate are obtained from FRED. To calibrate the employment wage premium, I follow Shimer (2005) in setting  $\gamma = 0.4$ . The relative efficiency is calibrated to  $\chi = 0.3$ , to match a monthly job-to-job transition rate of 3.2% in steady state, following Moscarini & Thomsson (2007).<sup>18</sup> Below, I discuss the implications of alternative parameterizations for  $\chi$  and  $\gamma$ .

**Results** The baseline estimates displayed in Figure 1 reveal rich dynamics in the welfare gains from employer entry over the 1999-2024 period. During expansionary phases,  $\mathcal{L}$  averages approximately 0.02, implying that a 1% increase in the mass of employers raises welfare by about 0.02% when the search parameters are held fixed. However,  $\mathcal{L}(M)$  increases sharply during economic downturns, reaching around 0.07 during the 2008-2009 financial crisis and exceeding 0.10 during the initial months of the 2020 pandemic recession.

<sup>18</sup>One can show that the average separation rate  $s$  satisfies  $s = \sigma + \lambda\chi \int_{w_\omega \geq w_r} [1 - F(w_\omega | w_r)] dG(w_\omega) = \sigma \left(1 + \frac{\sigma}{\lambda\chi}\right) \log\left(1 + \frac{\lambda\chi}{\sigma}\right)$ . Hence, given  $\lambda$  and  $\sigma$ , there exists a one-to-one mapping between the job-to-job transition rate  $s - \sigma$  and the relative efficiency of on-the-job search  $\chi$ .



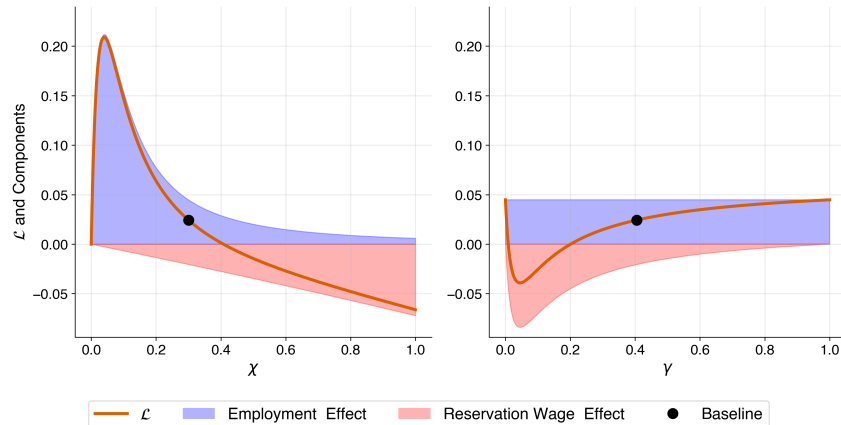


Figure 2: Sensitivity of  $\mathcal{L}(M)$  to  $\chi$  and  $\gamma$

*Note:* Sensitivity of the baseline estimates for the welfare gains from employer entry in the US labor market for homothetic search technologies, based on evaluating equation (29) at the average monthly US unemployment rate between 1999-2024. Estimates displayed in the left panel are for fixed  $\gamma = 0.4$ , and those in the right panel for fixed  $\chi = 0.3$ . Decomposition into extensive (blue) and intensive (red) margin effects based on the first and second summand on the right-hand-side of equation (29), respectively.

The decomposition into extensive and intensive margin effects illuminates the mechanisms driving these dynamics. The employment effect accounts for roughly two-thirds of  $\mathcal{L}$  during normal times, but becomes particularly dominant during recessions. This reflects how the marginal value of job creation rises when job-to-job transitions. In contrast, the reservation wage effect exhibits less cyclical variation and, hence, becomes proportionally smaller during recessions, explaining why the total gains from variety rise more than proportionally with higher unemployment. Importantly, reservation wage adjustments never result in negative returns to job creation because observed unemployment rates remain above the critical threshold (equal to  $u^*(\chi, \gamma) \approx 2.2\%$  at the baseline calibration) throughout the sample period.

**Sensitivity** Figure 2 explores how deviations from the baseline parameterization (shown as black dots) affect the welfare gains from job creation, holding unemployment fixed at its sample average.

The left panel demonstrates that reducing the relative search efficiency  $\chi$  below its baseline value of 0.302 initially amplifies the welfare gains from employer entry. Lower search efficiency makes existing employment relationships more persistent, which strengthens workers' incentives to accept job offers. However, for  $\chi$  below 0.2, this amplification exhibits strong diminishing returns. Conversely, raising above the

baseline gradually erodes the gains from job creation by intensifying reservation wage pressures, eventually driving the gains from job creation into negative territory. This pattern implies that policies aimed at facilitating job-to-job transitions may offer limited welfare benefits in labor markets where on-the-job search is already relatively efficient.

Following the earlier discussion, the employment wage premium  $\gamma$  shapes the gains from job creation solely through its impact on workers' perceived outside option value, as shown in the right panel of Figure 2. Around the baseline value of  $\gamma = 0.4$ , an increase in the employment wage premium ( $\gamma \downarrow$ ) weakens the welfare benefits of employer entry. This occurs because a larger gap between market and non-market returns increases the sensitivity of the reservation wage to employer entry in absolute value, thereby reducing the competitive benefits of having additional potential employers.

These quantitative findings underscore a broader insight: the welfare gains from employer entry are shaped by changes in the search technology in ways that would be difficult to discern from a purely preference-based approach. Specifically, the non-monotonic relationship between relative search efficiency and love-for-variety revealed in Figure 2 has important implications for labor market policies. Job search assistance programs or digital platforms that reduce matching frictions may actually decrease the welfare gains from job creation. Similarly, unemployment insurance design faces a delicate balancing act: while reducing benefit generosity decreases the relative value of non-employment (lowering  $\gamma$ ), my results show this can nevertheless weaken the welfare benefits of employer entry by amplifying reservation wage adjustment.

## 6 Concluding Remarks

This paper establishes a fundamental equivalence between search-theoretic and preference-based approaches to modeling labor market monopsony. By demonstrating that the canonical job ladder model maps into a representative agent framework with implicitly additive preferences, I show that the traditional dichotomy between "frictional" and "taste-driven" labor market imperfections may be less meaningful than previously thought.

The unified framework yields several key insights. First, it reveals that search frictions manifest as non-homothetic preferences over jobs, with the mean–min wage ratio emerging as a sufficient statistic for how wage inequality shapes the perceived

substitutability of employers. Second, it demonstrates that the classical distinction between voluntary and involuntary unemployment becomes theoretically ambiguous once the equivalence between search and preference-based approaches is recognized.

As one illustration of how this equivalence can be applied, I develop a measure of love-for-variety in the labor market. This application shows how the welfare gains from employer entry can be decomposed into extensive and intensive margin effects, with their relative importance varying systematically with labor market conditions. The countercyclical pattern in these gains, along with their sensitivity to search efficiency parameters, demonstrates how the unified framework can generate novel insights into labor market policies.

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# A Proofs

## A.1 Notation

In what follows, I implicitly index wage offers by  $\omega$ , via the mapping

$$w_\omega \equiv w(\omega).$$

From the definition of the wage offer distribution

$$F(w | w_r) = \frac{1}{|\Omega|} \int_{\Omega} \frac{\lambda_\omega}{\lambda} 1\{w_r \leq w_\omega \leq w\} d\omega,$$

where

$$\int_{\Omega} \frac{\lambda_\omega}{\lambda} d\omega = 1,$$

the two measures  $dw_\omega$  and  $d\omega$  are related by the Jacobian of the transformation  $w_\omega = w(\omega)$ . By the chain rule:

$$dw_\omega = \frac{dw_\omega}{d\omega} d\omega \equiv \frac{\lambda_\omega}{\lambda} d\omega,$$

or equivalently,

$$d\omega = \frac{dw_\omega}{\lambda_\omega/\lambda}.$$

This relation is used throughout the proofs; with a few typical applications highlight below:

- a. For a function  $h(w_\omega) = \int_{w_r}^{w_\omega} l(\xi) d\xi$ , differentiation with respect to  $\omega$  gives

$$\frac{dh(w_\omega)}{d\omega} = \frac{dh(w_\omega)}{dw_\omega} \cdot \frac{dw_\omega}{d\omega} = \frac{\lambda_\omega}{\lambda} = \frac{dh(w_\omega)}{dw_\omega}.$$

- b. When rewriting integrals involving the density  $n_\omega(w_\omega)$ , a factor of  $\lambda/\lambda_\omega$  naturally emerges. For example, consider an integral in  $\omega$ -space

$$\int \frac{\lambda}{\lambda_\omega} n_\omega(w_\omega) \left( 1 + \frac{\lambda\chi}{\sigma} \bar{F}(w_\omega | w_r) \right) d\omega$$

Here the ratio  $\lambda/\lambda_\omega$  arises from substituting

$$d\omega = \frac{dw_\omega}{\lambda_\omega/\lambda},$$

and later the definition of  $\bar{F}$  and related densities is set up so that these Jacobian factors cancel.

c. Integration by parts: Consider an integral in the wage-coordinate,

$$\int_{\Omega} l(w_{\omega}) \frac{dh(w_{\omega})}{dw_{\omega}} d\omega.$$

Rewriting in terms of  $w_{\omega}$  (by replacing  $d\omega = \frac{dw_{\omega}}{\lambda_{\omega}/\lambda}$ ) and then applying integration by parts (with the usual formula

$$\int u dv = uv - \int v du,$$

with appropriate boundary conditions) results in a transfer of the derivative from  $l$  to  $h$  and the appearance (and eventual cancellation) of the Jacobian factors.

In what follows I will often work with integrals expressed either in terms of  $w_{\omega}$  or  $\omega$  without further comment. It is understood that when converting between the two, one always uses

$$d\omega = \frac{dw_{\omega}}{\lambda_{\omega}/\lambda}.$$

## A.2 Characterization of the BM equilibrium

**Value functions** The value of unemployment  $U$  and the value of employment  $V(w)$  satisfy the Bellman equations

$$rU = \underline{w} + \lambda \int_{\Omega} [V(w_{\omega}) - U] dF(w_{\omega}|w_r), \quad (30)$$

and

$$rV(w) = w + \sigma [U - V(w)] + \lambda\chi \int_{\Omega} [V(w_{\omega}) - V(w)] dF(w_{\omega}|w), \quad (31)$$

where  $\lambda = \int_{\Omega} \lambda_{\omega} 1\{w^* \leq w_{\omega}\} d\omega$  and  $F(w_{\omega}|w) = \int_{\Omega} \frac{\lambda_{\omega}}{\lambda} 1\{w \leq w_{\omega'} \leq w_{\omega}\} d\omega$ .

From the indifference condition  $V(w_r) = U$ , the reservation wage hence satisfies:

$$w_r = \underline{w} + \lambda(1 - \chi) \int_{\Omega} [V(w_{\omega}) - V(w_r)] dF(w_{\omega}, w_r).$$



Integrating by parts after changing the variable of integration to  $dw_\omega = \frac{\lambda_\omega}{\lambda} d\omega$  obtains:

$$w_r = \underline{w} + \lambda(1 - \chi) \int_{\Omega} V'(w_\omega) \bar{F}(w_\omega | w_r) d\omega,$$

where  $\bar{F}(w_\omega | w_r) = 1 - F(w_\omega | w_r)$ . Differentiation of (31) with respect to  $dw_\omega$  yields

$$[r + \sigma + \lambda\chi(1 - F(w_\omega | w_r))] V'(w_\omega) = 1 \Rightarrow V'(w_\omega) = \frac{1}{r + \sigma + \lambda\chi\bar{F}(w_\omega | w_r)}$$

Substituting into the expression for the reservation wage yields:

$$w_r - b = \lambda(1 - \chi) \int_{\Omega} \frac{\bar{F}(w_\omega | w_r)}{r + \sigma + \lambda\chi(1 - F(w_\omega | w_r))} d\omega$$

**Non-employment rate** In a stationary equilibrium, worker flow balance into and out of non-employment implies:

$$u\lambda = (1 - u)\sigma \Leftrightarrow u = \frac{\sigma}{\lambda + \sigma}.$$

**Equilibrium wage distribution** To construct the wage distribution among employed workers,  $G(w)$ , impose that inflows and outflows balance at every wage interval  $[w^*, w)$ . Flow balance requires that

$$eG(w)\{\sigma + \lambda\chi[1 - F(w)]\} = u\lambda F(w).$$

where  $F(w) = \int_{\Omega} \frac{\lambda_\omega}{\lambda} 1\{w^* \leq w_\omega \leq w\} d\omega$  and  $\lambda = \int_{\Omega} \lambda_\omega d\omega$ . Solving for  $G(w)$  obtains:

$$G(w) = \frac{\sigma F(w)}{\sigma + \lambda\chi[1 - F(w)]}.$$

**Labor supply** Imposing flow balance at the firm-level, the labor supply to firm  $\omega$  equals

$$n_\omega = \frac{\lambda_\omega}{\lambda + \sigma} \frac{\sigma + \frac{\lambda\chi\sigma F(w_\omega | w_r)}{\sigma + \lambda\chi[1 - F(w_\omega | w_r)]}}{[\sigma + \lambda\chi[1 - F(w_\omega | w_r)]]} = \frac{\lambda_\omega}{\lambda + \sigma} \frac{1 + \chi\lambda/\sigma}{\left[1 + \frac{\lambda\chi}{\sigma} [1 - F(w_\omega | w_r)]\right]^2}$$

for  $w_\omega \geq w_r$ ; and 0 otherwise.

**The Average Wage** Denote  $W = \int_{\Omega} w_{\omega} dG(w_{\omega})$  the mean wage. Integration by parts then yields

$$\int_{\Omega} [1 - G(w_{\omega}|w_r)] d\omega = W - w_r \quad (32)$$

**Equilibrium and Aggregate Welfare** Given fundamentals  $\{(\lambda_{\omega})_{\omega \in \Omega}, \chi, \sigma, b\}$  and a vector of wage offers  $\mathbf{w}$ , a steady state equilibrium is a distribution  $G(w|w_r)$  such that the associated mean wage  $W = \int_{\Omega} w_{\omega} dG(w_{\omega}|w_r)$  and the reservation wage  $w_r$  jointly solve  $w_r - b = \frac{\lambda(1-\chi)}{\sigma+\lambda\chi} [W - w_r]$ . Summing over all agents, utilitarian welfare flow can then be written

$$ruV(w_r; \phi) + r \int_{\Omega} V(w_{\omega}, \phi) n(w_{\omega}; \phi) d\omega,$$

where the scaling  $r$  ensures that all values remain strictly positive and well-defined in the limiting case  $r/\lambda \rightarrow 0$ .

### A.3 Proof of Lemma 1

Letting  $r/\lambda \rightarrow 0$  implies that

$$\lim_{r/\lambda \rightarrow 0} \lambda V'(w) = \lim_{r/\lambda \rightarrow 0} \frac{1}{r/\lambda + \sigma/\lambda + \chi [1 - F(w|w_r)]} = \frac{1}{\sigma + \lambda\chi} \frac{1 - G(w|w_r)}{1 - F(w|w_r)}$$

Integration equation (30) by parts then yields the value of non-employment

$$\begin{aligned} \lim_{r/\lambda \rightarrow 0} rU &= \underline{w} + \lambda \int_{\Omega} V'(w_{\omega}|w_r) \bar{F}(w_{\omega}|w_r) d\omega \\ &= \underline{w} + \frac{\lambda}{\sigma + \lambda\chi} \int_{\Omega} [1 - G(y_{\omega}|w_r)] d\omega \\ &= \underline{w} + \frac{\lambda}{\sigma + \lambda\chi} (W - w_r) \end{aligned}$$

where the final line follows from (32). Similar steps yield the value of employment  $rV(w^*)$  at the reservation wage:

$$\lim_{r/\lambda \rightarrow 0} rV(w^*) = \frac{\sigma}{\sigma + \lambda\chi} w_r + \frac{\lambda\chi}{\sigma + \lambda\chi} W.$$

Setting  $U = V(w_r)$  yields  $w_r - \underline{w}$  as a function of  $W - w_r$

$$w_r - \underline{w} = \frac{\lambda(1-\chi)}{\lambda\chi + \sigma} [W - w_r]. \quad (33)$$

To characterize  $V(w)$  for general  $w > w_r$ , first note

$$V(x) - V(w) = \int_{w_r}^x V'(y) dy - \int_{w_r}^w V'(y) dy = \int_w^x V'(y) dy, \forall x \geq w$$

Using this to re-arrange equation (31) I obtain:

$$rV(w_\omega) = w - \sigma \int_{w_r}^{w_\omega} V'(w_\omega) d\omega + \lambda\chi \int_{w_\omega}^\infty \int_{w_\omega}^x V'(y) dy dF(x|w_r).$$

Changing the order of integration to rewrite the third summand

$$\int_{w_\omega}^\infty \int_{w_\omega}^x V'(y) dy dF(x) = \int_{w_\omega}^\infty V'(y) (1 - F(y|w_r)) dy,$$

and substituting back into  $rV(w_\omega)$ , I arrive at equation (14) in the main text.

$$\begin{aligned} \lim_{r/\lambda \rightarrow 0} rV(w) &= w - \lim_{r/\lambda \rightarrow 0} \left\{ \sigma \int_{w_r}^\infty V'(w) dw + \lambda\chi \int_w^\infty V'(y) \bar{F}(y) dy \right\} \\ &= w - \lim_{r/\lambda \rightarrow 0} \left\{ \sigma \int_{w_r}^\infty V'(x) dx + \int_w^\infty V'(y) [\lambda\chi \bar{F}(y) + \sigma] dy \right\} \\ &= w_r + \lim_{r/\lambda \rightarrow 0} \left\{ \int_{w^*}^\infty [1 - \sigma V'(x)] dx \right\} \\ &= w_r + \lambda\chi \int_{w_r}^\infty \frac{\bar{F}(w)}{\sigma + \lambda\chi \bar{F}(w)} dw \\ &= \frac{\sigma}{\sigma + \lambda\chi} w_r + \frac{\lambda\chi}{\sigma + \lambda\chi} W \equiv V(\mathbf{w}; \phi). \end{aligned}$$

Combining equations (33)-(14), yields  $w_r - \underline{w} = \frac{\lambda(1-\chi)}{\lambda\chi + \sigma} [W - w_r]$ , which in turn can be rewritten

$$\begin{aligned} \lim_{r/\lambda \rightarrow 0} w_r &= \underline{w} + \frac{\lambda(1-\chi)}{\sigma + \lambda\chi} \lim_{r/\lambda \rightarrow 0} [W - w_r] \\ &= \underline{w} + \frac{1-\chi}{\chi} \lim_{r/\lambda \rightarrow 0} [rV - w_r] \\ &= \chi \cdot \underline{w} + (1-\chi) \cdot V. \end{aligned}$$

Substituting into (14) shows that utilitarian welfare coincides with expected income:

$$V(\mathbf{w}; \phi) = \frac{\sigma}{\sigma + \lambda} \underline{w} + \frac{\lambda}{\sigma + \lambda} W(\mathbf{w}; \phi).$$

This completes the proof.

#### A.4 Proof of Lemma 2

I begin by manipulating the mean wage expression in (32).

$$\begin{aligned}
\int_{\Omega} [1 - G(w_{\omega}|w_r)] d\omega &= \frac{\sigma + \lambda\chi}{\sigma} \int_{\Omega} \frac{\bar{F}(w_{\omega}|w_r)}{1 + \frac{\lambda\chi}{\sigma}\bar{F}(w_{\omega}|w_r)} d\omega \\
&= \frac{1}{e} \int_{\Omega} n_{\omega}(w_{\omega}) \left(1 + \frac{\lambda\chi}{\sigma}\bar{F}(w_{\omega}, w_r)\right) \frac{d\omega}{\lambda/\lambda_{\omega}} \\
&= \int_{\Omega} \left(1 + \frac{\lambda\chi}{\sigma}\bar{F}(w_{\omega}|w_r)\right) \bar{F}(w_{\omega}|w_r) \frac{d \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi, \omega)}{e} d\xi \right]}{dw_{\omega}} dw_{\omega} \\
&= - \int_{\Omega} \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi, \omega)}{e} d\xi \right] d \left[ \left(1 + \frac{\lambda\chi}{\sigma}\bar{F}(w_{\omega})\right) \bar{F}(w_{\omega}) \right] \\
&= \int_{\Omega} \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi)}{e} d\xi \right] \left(1 + 2\frac{\lambda\chi}{\sigma}\bar{F}(w_{\omega})\right) d\omega,
\end{aligned}$$

Here, the second line obtains from substituting equation (12). The third line follows from using the definition of  $F$  and differentiating the cumulative density  $\int_{w_r}^{w_{\omega}} \frac{n(\xi)}{e} d\xi$ , with respect to  $d\omega \equiv \frac{\lambda_{\omega}}{\lambda} dw_{\omega}$ . The fourth line results from a change in the order of integration. The fifth line applies integration by parts under the limits  $\lim_{w \rightarrow \infty} \bar{F}(w|w_r) \int_{w_r}^w n(\xi) d\xi = \lim_{w \rightarrow w_r} \bar{F}(w|w_r) \int_{w_r}^w n(\xi) d\xi = 0$  and then changes back the variable of integration by setting:  $\frac{\lambda_{\omega}}{\lambda} dw_{\omega} = d\omega$ . Note that differentiation with respect to  $w_{\omega}$  introduces a factor  $\lambda_{\omega}/\lambda$  which is then cancelled when changing back the variable of integration to  $\omega$ . Therefore, the final expression is written solely in terms of  $d\omega$  without any residual  $\lambda_{\omega}$  factors.

Further manipulation reveals that

$$\begin{aligned}
2 \int_{\Omega} \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi_{\omega})}{e} d\xi \right] \frac{\lambda\chi}{\sigma} \bar{F}(w_{\omega}) d\omega &= \frac{2\lambda\chi}{\sigma} \int_{w_r}^{\infty} \frac{n(\xi_{\omega})}{e} \left[ \int_{F(w_{\omega})}^1 [1-y] dy \right] d\omega \\
&= \frac{\lambda\chi}{\sigma} \int_{w_r}^{\infty} \frac{n(\xi_{\omega})}{e} \bar{F}(w_{\omega})^2 d\omega \\
&= \frac{\lambda\chi}{\lambda\chi + \sigma} \int_{w_r}^{\infty} \left[ \frac{(\sigma + \lambda\chi) \bar{F}(w_{\omega}|w_r)}{\sigma + \lambda\chi \bar{F}(w_{\omega}|w_r)} \right]^2 d\omega \\
&= \frac{\lambda\chi}{\lambda\chi + \sigma} \int_{w_r}^{\infty} [1 - G(w)]^2 dw \\
&= \frac{\lambda\chi}{\lambda\chi + \sigma} \{W - w_r - WG(G)\}.
\end{aligned}$$

where

$$\mathcal{G}(G) \equiv \frac{1}{W} \int_{w_r}^{\infty} [1 - G(w)] G(w) dw.$$

is the Gini coefficient of  $G$ . Here, the first line uses Fubini's theorem to switch the order between the  $\xi$  and  $\omega$  integrations, re-expressing the inner term as the cumulative density associated with the survival function  $\bar{F} = 1 - F$ , which can be rewritten as  $\int_{F(w_{\omega})}^1 [1 - y] dy$ . The third line evaluates the cumulative density of  $\bar{F}$  and substitutes substitutes the definition of  $n_{\omega}$  in (12). The fifth line changes the variable of integration and the fifth line follows from integration by parts and (32).

Combine these expressions with (32) to obtain

$$\frac{W}{w_r} \left( 1 + \frac{\lambda\chi}{\sigma} \mathcal{G}_G \right) = 1 + \left( 1 + \frac{\lambda\chi}{\sigma} \right) \int_{\Omega} \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi)}{1-u} d\xi \right] d\omega, \quad (34)$$

Substitution into (14) then obtains:

$$\begin{aligned}
\mathcal{V} &= \left( \frac{\sigma}{\sigma + \lambda\chi} + \frac{\frac{\lambda\chi}{\sigma + \lambda\chi} + \frac{\lambda\chi}{\sigma} \int_{\Omega} \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi)}{e} d\xi \right] d\omega}{\left( 1 + \frac{\lambda\chi}{\sigma} \mathcal{G}_G \right)} \right) w_r \\
\Leftrightarrow 1 + \frac{\lambda\chi}{\sigma} \mathcal{G}_G &= \frac{w_r}{\mathcal{V}} \frac{\frac{\lambda\chi}{\sigma + \lambda\chi} + \frac{\lambda\chi}{\sigma} \int_{\Omega} \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi)}{e} d\xi \right] d\omega}{1 - \frac{\sigma}{\sigma + \lambda\chi} \frac{w_r}{\mathcal{V}}}
\end{aligned}$$

Now, define  $\alpha$  to satisfy  $\frac{W}{w_r} = \frac{W}{\mathcal{V}} \frac{\mathcal{V}}{w_r} = \frac{W}{\frac{\sigma}{\sigma + \lambda\chi} w_r + \frac{\lambda}{\lambda + \sigma} W} \frac{\mathcal{V}}{w_r} \equiv \frac{1}{\alpha} \frac{\mathcal{V}}{w_r}$ . Then, (34) simplifies to:

$$1 + \frac{\lambda\chi}{\sigma} \mathcal{G}_G = \alpha \frac{w_r}{\mathcal{V}} \left( 1 + \left( 1 + \frac{\lambda\chi}{\sigma} \right) \int_{\Omega} \left[ \int_{w_r}^{w_{\omega}} \frac{n(\xi)}{e} d\xi \right] d\omega \right),$$

Combining with the expression for welfare above then obtains:

$$\int_{\Omega} \left[ \int_{w_r/\mathcal{V}}^{w_{\omega}/\mathcal{V}} \frac{n(\xi, w_r/V)}{1-u} d\xi \right] d\omega = \frac{w_r}{\mathcal{V}} \frac{\frac{\lambda\chi}{\sigma+\lambda\chi} - \alpha \left(1 - \frac{w_r}{V} \frac{\sigma}{\sigma+\lambda\chi}\right)}{\frac{\lambda\chi}{\sigma} - \left(1 + \frac{\lambda\chi}{\sigma}\right) \alpha \left(1 - \frac{w_r}{V} \frac{\sigma}{\sigma+\lambda\chi}\right)} = \frac{w_r}{\mathcal{V}} \frac{1}{1 + \frac{\lambda\chi}{\sigma}}$$

From a change in the variable of integration to  $d\zeta = \frac{V}{w_r} d\xi$ , I then finally arrive at

$$\frac{1-u}{1 + \frac{\lambda\chi}{\sigma}} = \int_{\Omega} \left[ \int_1^{w_{\omega}/w_r} n_{\omega}(\xi, 1) d\zeta \right] d\omega,$$

Now, divide both sides of the above equation  $1-u$  and, change the variable of integration to  $d\zeta = \frac{1}{1-u} d\xi$ . Then, noting that  $n_{\omega}$  in equation (12) is homogeneous of degree 0 in  $w_{\omega}$  and  $w_r$ , I can define  $\ell_{\omega}(\xi_{\omega}) \equiv n_{\omega}(\frac{w_{\omega}}{w_r}, 1/e)$  to arrive at equation (17) in the main text. This completes the proof.

## A.5 Proof of Lemma 3

I begin by re-arranging the expressions in Lemma 1 to obtain the aggregate inequality measure  $\frac{W}{w_r}$  as a function of  $\frac{W}{w}$  and search parameters:

$$\frac{w}{w_r} = \frac{1 - (1-\chi)e\Lambda}{\chi + (1-e)(1-\chi)}.$$

Using this to rewrite equation (14), welfare can then be written

$$\mathcal{V} = w_r \left( \frac{\sigma}{\sigma + \lambda\chi} + \frac{\lambda\chi}{\sigma + \lambda\chi} K(\Lambda) \right) \equiv w_r L(\Lambda) \equiv eW \frac{L(\Lambda)}{K(\Lambda)} \frac{\sigma + \lambda}{\lambda},$$

where I defined

$$\frac{W}{w_r} \equiv K(\Lambda) \equiv \frac{\Lambda}{1 + (1-\chi) \frac{\lambda}{\lambda+\sigma} (\Lambda - 1)}, \quad \text{for } \Lambda \equiv \frac{W}{w}.$$

Now, multiply and divide the upper bound of the inner integral in (17) by  $W$  to express Lemma 2 in terms of total employment income and  $K(\Lambda)$ :

$$\frac{\sigma}{\sigma + \lambda\chi} = \int_{\Omega} \int_{1/e}^{\frac{w_{\omega}}{Y(w,w)} K(\frac{W}{w})} \ell_{\omega}(\xi) d\omega, \quad (35)$$

where  $eW = Y(\mathbf{w}, \underline{w}) = \int_{\Omega} n_{\omega} w_{\omega} d\omega$ . Now, since  $Y$  is linearly homogeneous in  $(\mathbf{w}, \underline{w})$  and  $\ell_{\omega}$  is monotonic, the above condition allows me to define a function

$$H(\mathbf{w}, \Lambda) = \int_{\Omega} \int_{1/e}^{\frac{w_{\omega}}{Y(\mathbf{w}, \Lambda)} K(\Lambda)} \ell_{\omega}(\xi) d\omega - \frac{\sigma}{\sigma + \lambda \chi}$$

so that the implicit function theorem implies the existence of a unique, continuously differentiable function  $\Lambda(\mathbf{w})$  satisfying

$$H\left(\mathbf{w}, \Lambda\left(\frac{\mathbf{w}}{\underline{w}}\right)\right) = 0.$$

Because the above holds identically for each  $\mathbf{w}$ , any small change in  $\Lambda$  must be exactly offset by the structure of the function so that the condition continues to hold. Differentiating with respect to  $\Lambda$ , holding  $\mathbf{w}$  fixed, thus gives

$$\left. \frac{d}{d\Lambda} \frac{K(\Lambda)}{Y(\mathbf{w}, \Lambda)} \right|_{\Lambda=\Lambda(\mathbf{w})} = 0.$$

Now, use the definitions laid out earlier to express welfare and  $Y/K$  as functions of  $Y$  and  $\frac{W}{\underline{w}}$ :

$$\mathcal{V} = \frac{W}{K(\frac{W}{\underline{w}})} L\left(\frac{W}{\underline{w}}\right) = \frac{1}{e} \frac{Y}{K(\frac{W}{\underline{w}})} L\left(\frac{W}{\underline{w}}\right),$$

and

$$\frac{1}{e} \frac{Y\left(\mathbf{w}, \frac{W}{\underline{w}}\right)}{K(\frac{W}{\underline{w}})} = \chi + (1 - \chi) L\left(\frac{W}{\underline{w}}\right).$$

Re-arrangement of the last expression yields:

$$L\left(\frac{W}{\underline{w}}\right) = \frac{1}{1 - \chi} \left( \frac{1}{e} \frac{Y\left(\mathbf{w}, \frac{W}{\underline{w}}\right)}{K(\frac{W}{\underline{w}})} - \chi \right).$$

Substituting into welfare, I then obtain:

$$\mathcal{V} = \frac{1}{e} \frac{Y(\mathbf{w}, \Lambda(\mathbf{w}))}{K(\Lambda(\mathbf{w}))} \frac{1}{1 - \chi} \left( \frac{1}{e} \frac{Y(\mathbf{w}, \Lambda(\mathbf{w}))}{K(\Lambda(\mathbf{w}))} - \chi \right).$$

Since the only  $\Lambda$ -dependence in  $\mathcal{V}$  comes through the ratio  $\frac{Y(\mathbf{w}, \Lambda(\mathbf{w}))}{K(\Lambda(\mathbf{w}))}$ , the fact that

$$\left. \frac{d}{d\Lambda} \frac{K(\Lambda)}{Y(\mathbf{w}, \Lambda)} \right|_{\Lambda=\Lambda(\mathbf{w})} = 0,$$

implies that

$$\frac{\partial \mathcal{V}}{\partial \Lambda} = 0.$$

Expressed in logs, this is equivalent to:

$$\frac{\partial \ln Y}{\partial \ln \frac{W}{w}} + \frac{\partial \ln \frac{L}{K}}{\partial \ln \frac{W}{w}} = 0.$$

Differentiating  $\mathcal{V}$  with respect to  $w_\omega$  and  $Y$  then yields:

$$\frac{\partial \mathcal{V}}{\partial w_\omega} = \frac{L(\Lambda)}{K(\Lambda)} \frac{\ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right)}{\int_\Omega w_\omega \ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right) d\omega} Y,$$

and

$$\frac{d\mathcal{V}}{dY} = \frac{L(\Lambda)}{K(\Lambda)}.$$

Noting that  $dY = de_{w\Lambda}$ , I arrive at the statement given in Lemma 3:

$$\frac{\frac{\partial \mathcal{V}}{\partial w_\omega}}{\frac{\partial \mathcal{V}}{\partial Y}} = \frac{\ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right)}{\int_\Omega \frac{w_\omega}{eW} \ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right) d\omega}.$$

This completes the proof.

## A.6 Proof of Theorem 1

Now, consider a representative agent problem of the following form:

$$V(\mathbf{w}, C) = \arg \max_{\mathbf{n}} U(C, \mathbf{n}) + \nu \left( C - \frac{\sigma}{\sigma + \lambda} w - \int n_\omega w_\omega d\omega \right)$$

The FOC's are:

$$\begin{aligned} \frac{\partial U}{\partial C} &= \nu. \\ -w_\omega \frac{\partial U}{\partial n_\omega} &= \nu n_\omega w_\omega = \frac{\partial U}{\partial C} n_\omega w_\omega \end{aligned}$$



Aggregating:

$$\frac{\partial U}{\partial C} \int_{\Omega} n_{\omega} w_{\omega} d\omega = - \int \frac{\partial U}{\partial n_{\omega}} w_{\omega} d\omega.$$

$$n_{\omega} = \frac{\frac{\partial U}{\partial n_{\omega}}}{\int \frac{w_{\omega}}{n_{\omega} w_{\omega}} \frac{\partial U}{\partial n_{\omega}} d\omega}$$

Then, the envelope theorem yields:

$$\frac{\partial \ln V}{\partial \ln w_{\omega}} = \frac{w_{\omega}}{V} \frac{\partial U}{\partial C} n_{\omega} = \frac{\partial \ln V}{\partial \ln Y} w_{\omega} n_{\omega}$$

And thus:

$$\nu = \frac{\partial \ln U}{\partial \ln C} = \int_{\Omega} \frac{\partial \ln V}{\partial \ln w_{\omega}} d\omega$$

These equations allow to recover  $U$ . Now, the aggregate welfare function defined in the proof of Lemma 3 clearly satisfies these conditions. To verify that  $\mathcal{V} = W(\mathbf{w}, \underline{w}) \frac{L(\frac{W}{w})}{K(\frac{W}{w})}$  is indeed an indirect utility function, I verify that it satisfies Assumption 1 for  $\ell_{\omega}$  given by Lemma 2 and

$$L(\Lambda) \equiv \frac{\sigma}{\sigma + \lambda\chi} + \frac{\lambda\chi}{\sigma + \lambda\chi} K(\Lambda),$$

and

$$K(\Lambda) \equiv \frac{\Lambda}{1 + (1 - \chi) \frac{\lambda}{\sigma + \lambda} (\Lambda - 1)} > 1.$$

It is obvious that  $K$ ,  $L$ , and  $\{\ell_{\omega}\}_{\omega \in \Omega}$  are all strictly positive, continuously differentiable functions. Differentiation of  $\ell_{\omega}$  yields

$$\varepsilon_{\ell, \omega} = 2 \frac{\lambda\chi}{\sigma} \frac{z_{\omega} F'(z_{\omega})}{1 + \frac{\lambda\chi}{\sigma} [1 - F(z_{\omega})]} > 0, \quad \forall z_{\omega} > 1 + \frac{\lambda}{\sigma},$$

where the inequality follows from the fact that  $F$  is a cdf with no mass points on the interior of its domain  $(1 + \frac{\lambda}{\sigma}, \infty)$ . Moreover, differentiation of  $K$  and  $L$  with respect to  $\Lambda$  reveals that  $(\frac{\partial \ln L/K}{\partial \ln \Lambda} + \varepsilon_{\ell, \omega} \frac{\partial \ln L}{\partial \ln \Lambda}) \frac{w_{\omega} \ell_{\omega}(z_{\omega})}{\int_{\Omega} w_{\omega} \ell_{\omega}(z_{\omega}) d\omega} + \varepsilon_{\ell, \omega} > -1$  if, and only if,

$$B \equiv \frac{1 - e}{1 + e(\Lambda - 1)} \left( \frac{\Lambda (1 + \varepsilon_{\ell, \omega})}{\Lambda + \frac{\sigma}{\lambda\chi} [1 + e(\Lambda - 1)]} - 1 \right) m_{\omega} + \varepsilon_{\ell, \omega} > -1.$$

where I denoted  $m_\omega \equiv \frac{w_\omega \ell_\omega(z_\omega)}{\int_\Omega w_\omega \ell_\omega(z_\omega) d\omega}$ . Noticing that  $B$  is strictly increasing in  $\varepsilon_{\ell, \omega}$  and that  $\Lambda \geq 1$ ,  $e < 1$  and  $m_\omega \in [0, 1]$  by definition, the term  $B$  can be bounded below as follows:

$$\begin{aligned} B &\geq \frac{1-e}{1+e(\Lambda-1)} \left( \underbrace{\frac{\Lambda}{\Lambda + \frac{\sigma}{\lambda\chi} [1 + e(\Lambda-1)]}}_{\in (0,1)} - 1 \right) m_\omega \\ &\geq - \underbrace{\frac{1-e}{1+e(\Lambda-1)}}_{<1-e<1} \underbrace{m_\omega}_{\leq 1} \\ &> -1. \end{aligned}$$

Finally, because  $\ell_\omega$  is increasing on its entire domain and bounded strictly away from 0, the function

$$A(Y; \mathbf{w}, \underline{w}) = \int_\Omega w_\omega \ell_\omega \left( \frac{w_\omega}{Y} K\left(\frac{Y}{e\underline{w}}\right) \right) d\omega - Y$$

satisfies  $A'(Y) < 0$  for all  $Y \in (\underline{w}, \infty)$ , as well as

$$\lim_{Y \rightarrow \underline{w}} A(Y) > 0,$$

and

$$\lim_{Y \rightarrow \infty} A(Y) < 0,$$

for all  $\mathbf{w} = \{w_\omega \geq \underline{w} : \omega \in \Omega\}$  and  $\underline{w} > 0$ . By the Intermediate value theorem, there hence exists a unique  $Y^* \in (0, \infty)$  s.t.  $A(Y^*) = 0$ . Consequently, the function  $\mathcal{V}$  defined in Theorem 1 satisfies Assumption 1 and, thus, characterizes a well-behaved utility. This completes the proof.

## A.7 Proof of Corollary 2

When  $\underline{w} \equiv \gamma W$  for  $\gamma < 1$ , then Theorem 1 implies that  $K(\Lambda)$  is equal to a constant, given by

$$\frac{W}{w_r} = K(\Lambda) = \frac{\gamma}{\gamma - e(1-\chi)(1-\gamma)} > 1.$$

This implies that the ratio of welfare to the reservation wage can be written as

$$\frac{\mathcal{V}}{w_r} = \frac{\mathcal{V}}{W} \frac{W}{w_r} = \frac{\gamma\sigma + \lambda}{\gamma\sigma + \lambda [1 - \chi(1-\gamma)]}.$$

Re-arrangement of (17) then yields

$$\int_{\Omega} \left[ \int_{\underline{z}}^{w_{\omega}/\mathcal{V}} \ell_{\omega}(\xi, \underline{z}) d\xi \right] d\omega = \underline{z},$$

with  $\ell_{\omega}(\cdot)$  defined as in Lemma 2 and

$$\underline{z} \equiv \frac{w_r}{\mathcal{V}} = \frac{\gamma + \frac{\lambda}{\sigma} [1 - \chi(1 - \gamma)]}{\gamma + \frac{\lambda}{\sigma}}$$

This completes the proof.

## A.8 Proof of Proposition 1

Assume that the indirect utility of worker  $i$  when choosing firm  $\omega$  is given by

$$u_i(\omega; \phi) = v(\omega; \phi) + \epsilon_i(\omega),$$

where  $v(\omega, \phi) \equiv \ln \left[ w_{\omega} \ell_{\omega} \left( \frac{w_{\omega}}{\mathcal{V}}; \phi \right) \right]$  and  $\epsilon_i(\omega)$  are i.i.d. Gumbel random variables with scale parameter  $\mu$ . Under these conditions the choice probability of employer  $\omega$  is given by the multinomial logit formula:

$$\mathbb{P}[\omega; \phi] = \mathbb{P} \left[ \omega = \arg \max_{\omega'} u_i(\omega; \phi) \right] = \frac{\exp \left( \frac{v(\omega; \phi)}{\mu} \right)}{\int_{\omega' \in \Omega} \exp \left( \frac{v(\omega'; \phi)}{\mu} \right) d\omega'}.$$

Substituting the expression for  $u_i(\omega; \phi)$  into the above yields

$$\mathbb{P}(\omega; \phi) = \frac{\exp \left( \ln \left[ w_{\omega} \ell_{\omega} \left( \frac{w_{\omega}}{\mathcal{V}}; \phi \right) \right] \right)}{\sum_{\omega' \in \Omega} \exp \left( \ln \left[ w_{\omega'} \ell_{\omega'} \left( \frac{w_{\omega'}}{\mathcal{V}}; \phi \right) \right] \right)} = \frac{w_{\omega} \ell_{\omega} \left( \frac{w_{\omega}}{\mathcal{V}}; \phi \right)}{\sum_{\omega' \in \Omega} w_{\omega'} \ell_{\omega'} \left( \frac{w_{\omega'}}{\mathcal{V}}; \phi \right)}$$

.The right-hand side of this expression implies the market share function

$$m_{\omega} \left( \frac{w_{\omega}}{\mathcal{V}}; \phi \right) \equiv \frac{w_{\omega} \ell_{\omega} \left( \frac{w_{\omega}}{\mathcal{V}}; \phi \right)}{\int_{\Omega} w_{\omega'} \ell_{\omega'} \left( \frac{w_{\omega'}}{\mathcal{V}}; \phi \right) d\omega'}$$

where

$$\int_{\Omega} m_{\omega} \left( \frac{w_{\omega}}{\mathcal{V}}; \phi \right) d\omega = 1.$$

This obviously coincides with the labor supply system generated by the homothetic search technology  $\phi$  defined in Corollary 2, and hence completes the proof.

## A.9 Proof of Proposition 2

I begin with the normalized offer--density function for each firm  $\omega$ :

$$\int_{\underline{z}}^{z_{\omega}} \ell_{\omega}(z_{\omega}; \phi) = \int_{\underline{z}}^{z_{\omega}} \frac{\lambda_{\omega}}{\lambda} \frac{\left(1 + \frac{\lambda\chi}{\sigma}\right)^2}{\left(1 + \frac{\lambda\chi}{\sigma} [1 - F(\xi | \underline{z})]\right)^2} d\xi$$

Under wage and contact rate symmetry,

$$\int_{\underline{z}}^{z_{\omega}} \ell_{\omega}(z_{\omega}; \phi) = \frac{1}{M} \left(1 + M \frac{\lambda_0\chi}{\sigma}\right)^2 (z - \underline{z}(\phi)),$$

for all  $\omega \in \Omega$ . Then, equation (24) implies:

$$\left(1 + M \frac{\lambda_0\chi}{\sigma}\right)^2 \left(\frac{1}{\mathcal{V}} - \underline{z}(\phi)\right) = \underline{z}(\phi).$$

Solving for  $\mathcal{V}$  yields:

$$\mathcal{V}(\mathbf{1}_{\Omega}) = \frac{1}{\underline{z}(M)} \frac{\left(1 + M \frac{\lambda_0\chi}{\sigma}\right)^2}{\left(1 + M \frac{\lambda_0\chi}{\sigma}\right)^2 + 1},$$

where

$$\underline{z}(M) \equiv \frac{\gamma + \frac{\lambda_0 M}{\sigma} [1 - \chi(1 - \gamma)]}{\gamma + \frac{\lambda_0 M}{\sigma}}.$$

Log-differentiation of  $V(\mathbf{1}_{\Omega})$  with respect to  $M$  yields (29) from the main text:

$$\mathcal{L}(M) = \chi(1 - u) [\mathcal{L}_E(u, \chi) - \mathcal{L}_I(u, \chi, \gamma)].$$

where I denoted

$$\mathcal{L}_E \equiv \frac{2}{[u + \chi(1 - u)] \left(1 + \left(1 + \frac{\chi(1 - u)}{u}\right)^2\right)},$$

$$\mathcal{L}_I \equiv \frac{u(1-\gamma)\gamma}{[\gamma + (1-\gamma)u] [\gamma + (1-\chi)(1-\gamma)u]}.$$

### A.10 Proof of Proposition 3

As  $u \rightarrow 0^+$ , then  $\mathcal{L}(u, \chi, \gamma) \approx \frac{2u^2}{\chi^3} - (\frac{1}{\gamma} - 1)u = -(\frac{1}{\gamma} - 1)u + O(u^2) < 0$ , so  $\mathcal{L}(u, \chi, \gamma)$  is negative for sufficiently small  $u > 0$ . By definition, the right derivative  $\mathcal{L}'(0)$  is exactly this limit, hence

$$\mathcal{L}'(0) = \lim_{u \rightarrow 0^+} \frac{\mathcal{L}(u) - \mathcal{L}(0^+)}{u - 0} = \lim_{u \rightarrow 0^+} \frac{\mathcal{L}(u)}{u} = -\chi(\frac{1}{\gamma} - 1) < 0.$$

As  $u \rightarrow 1$ ,  $\mathcal{L} \rightarrow 0$ . Moreover, setting  $v = 1 - u$ , so that  $v \rightarrow 0^+$  as  $u \rightarrow 1^-$ , the one-sided derivative at  $u = 1$  is given by

$$\mathcal{L}'(1) = \lim_{u \rightarrow 1^-} \frac{\mathcal{L}(1) - \mathcal{L}(u)}{1 - u} = - \lim_{u \rightarrow 1^-} \frac{\mathcal{L}(u)}{1 - u}.$$

Since  $1 - u = e$ , this is

$$\mathcal{L}'(1) = - \lim_{e \rightarrow 0^+} \frac{\chi e [\mathcal{L}_E(1 - e, \chi) - \mathcal{L}_I(1 - e, \chi, \gamma)]}{e} = -\chi \lim_{e \rightarrow 0^+} \mathcal{L}(1 - e, \chi, \gamma).$$

Since  $\lim_{v \rightarrow 0^+} \mathcal{L}(1 - e, \chi, \gamma) = 1 - \frac{(1-\gamma)}{[1+(\frac{1}{\gamma}-1)(1-\chi)]} > 0$ , this implies that  $\mathcal{L}'(u = 1) < 0$ . From the intermediate value theorem, it follows that  $\exists u^* \in (0, 1)$  so that  $\mathcal{L}(u^*) = 0$ ,  $\mathcal{L}'(u^*) > 0$ .

To characterize the derivatives, let

$$\Phi(u, \chi, \gamma) = \mathcal{L}(u, \chi, \gamma) = 0,$$

so that  $u^*$  is characterized by  $\Phi(u^*, \chi, \gamma) = 0$ . Provided

$$\frac{\partial \Phi}{\partial u}(u^*, \chi, \gamma) \neq 0,$$

the Implicit Function Theorem then guarantees that there is a neighborhood of  $(\chi, \gamma)$  in which  $u^*$  can be uniquely expressed as a continuously differentiable function of  $(\chi, \gamma)$ .

Moreover, the partial derivatives of  $u^*$  with respect to  $\chi$  and  $\gamma$  satisfy

$$\frac{\partial u^*}{\partial \chi} = -\frac{\frac{\partial \Phi}{\partial \chi}(u^*, \chi, \gamma)}{\frac{\partial \Phi}{\partial u}(u^*, \chi, \gamma)}, \quad \frac{\partial u^*}{\partial \gamma} = -\frac{\frac{\partial \Phi}{\partial \gamma}(u^*, \chi, \gamma)}{\frac{\partial \Phi}{\partial u}(u^*, \chi, \gamma)}.$$

From here, inspection of  $\mathcal{L}$  reveals that  $\frac{\partial \Phi}{\partial u}(u^*, \chi, \gamma) = \frac{\partial \mathcal{L}_E - \partial \mathcal{L}_I}{\partial u} \Big|_{\mathcal{L}_E = \mathcal{L}_I} < 0$ ,  $\frac{\partial \Phi}{\partial \chi}(u^*, \chi, \gamma) < 0$  and  $\frac{\partial \Phi}{\partial \gamma}(u^*, \chi, \gamma) < 0$ , which completes the proof.

## B [Online Appendix] Derivations for the Representative Agent Model

### B.1 Labor Supply

Equations (1)-(3) in the main text imply that the representative agent's indirect utility can be written as

$$\mathcal{V} := \mathcal{V}(Y(\mathbf{w}, \Lambda), \Lambda), \quad \text{where } \Lambda = \frac{W}{w}.$$

Differentiation of  $\mathcal{V}$  with respect to  $Y$  and  $\Lambda$  yields, respectively

$$\frac{d\mathcal{V}}{dY} = \frac{L(\Lambda)}{K(\Lambda)},$$

since the envelope condition in (3) implies that

$$\frac{\partial \ln \mathcal{V}}{\partial \ln \Lambda} = \frac{\partial \ln Y/K(\Lambda)}{\partial \ln \Lambda} + \frac{\partial \ln L}{\partial \ln \Lambda} = 0,$$

Moreover,

$$\frac{\partial \mathcal{V}}{\partial w_\omega} = \frac{L(\Lambda)}{K(\Lambda)} \frac{\partial Y}{\partial w_\omega} + \frac{\partial \mathcal{V}}{\partial \Lambda} \frac{\partial \Lambda}{\partial w_\omega} = \frac{\partial Y}{\partial w_\omega}.$$

Differentiating (2) with respect to  $\mathbf{w}$  attains the derivative of  $Y$  with respect to  $w_\omega$

$$\frac{\partial Y(\mathbf{w}, \Lambda)}{\partial w_\omega} = \frac{\ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right)}{\int_\Omega \frac{w_\omega}{Y} \ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right) d\omega},$$

Combining with the partial derivatives of  $\mathcal{V}$  shows that the labor supply system for firms  $\omega \in \Omega$  satisfies

$$\frac{\frac{\partial \mathcal{V}}{\partial w_\omega}}{\frac{\partial \mathcal{V}}{\partial Y}} = \frac{\ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right)}{\int_\Omega \frac{w_\omega}{Y} \ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right) d\omega},$$

so that

$$\int_\Omega w_\omega \frac{\frac{\partial \mathcal{V}}{\partial w_\omega}}{\frac{\partial \mathcal{V}}{\partial Y}} d\omega = Y.$$

## B.2 Properties of $\mathcal{V}$

Here, I show that the assumptions stated in main text are sufficient to guarantee that  $\mathcal{V}$  represents a well-behaved preference over firm-level labor allocations.

**Continuity and Differentiability** Given that  $K, L, \ell_\omega \in \mathcal{C}^2$  by assumption, the composite function theorem immediately implies that  $\mathcal{V} \in \mathcal{C}^2$ .

**Monotonicity** Since the functions  $K, L, \ell_\omega$  all have a strictly positive range, the derivations in the previous subsection immediately imply that  $\frac{\partial \mathcal{V}}{\partial w_\omega} > 0$  whenever  $w_\omega \geq zY/K(\Lambda)$ . To check monotonicity with respect to  $\underline{w}$ , note that the envelope condition implies that

$$\frac{\partial \ln Y}{\partial \ln \Lambda} = \frac{\partial \ln K(\Lambda)}{\partial \ln \Lambda} - \frac{\partial \ln L(\Lambda)}{\partial \ln \Lambda}$$

**Quasi-convexity in  $\mathbf{w}$**  Denote  $\varepsilon_{\ell, \omega} = \frac{\partial \ln \ell_\omega}{\partial \ln z_\omega}$ . Using the envelope condition to totally differentiate  $\frac{\partial \mathcal{V}}{\partial w_\omega} = \frac{L(\Lambda)}{K(\Lambda)} \ell_\omega \left( \frac{w_\omega}{Y} K(\Lambda) \right)$ , I obtain:

$$\begin{aligned} d \ln w_\omega \frac{\partial \mathcal{V}}{\partial w_\omega} &= d \ln w_\omega + d \ln \frac{L(\Lambda)}{K(\Lambda)} + \varepsilon_{\ell, \omega} \left( d \ln w_\omega + d \ln \frac{K(\Lambda)}{Y(\Lambda)} \right) \\ &= d \ln w_\omega + d \ln \frac{L(\Lambda)}{K(\Lambda)} + \varepsilon_{\ell, \omega} (d \ln L(\Lambda) + d \ln w_\omega). \end{aligned}$$

Since  $\partial \Lambda / \partial w_\omega = \frac{1}{\underline{e} \underline{w}} \frac{\partial Y}{\partial w_\omega} > 0$ , a sufficient condition for  $\frac{d \ln w_\omega \frac{\partial \mathcal{V}}{\partial w_\omega}}{d \ln w_\omega} > 0$  is given by

$$\left( \frac{\partial \ln L/K}{\partial \ln \Lambda} + \varepsilon_{\ell, \omega} \frac{\partial \ln L}{\partial \ln \Lambda} \right) \frac{w_\omega \ell_\omega(z_\omega)}{\int_\Omega w_\omega \ell_\omega(z_\omega) d\omega} + 1 + \varepsilon_{\ell, \omega} > 0, \quad \text{and} \quad \varepsilon_{\ell, \omega} > 0,$$

for all  $\omega, \Lambda$  and  $z_\omega \geq \underline{z}$ . By imposing both of these conditions, Assumption 1 hence guarantees the quasi-convexity of  $\mathcal{V}$  in  $\mathbf{w}$ .

**Uniqueness** The second part of Assumption 1 ensures that for all  $\mathbf{w}$  and  $\underline{w}$ , there exists a unique  $Y^*(\mathbf{w}, \underline{w})$  such that

$$Y^*(\mathbf{w}, \underline{w}) = \int_{\Omega} w_{\omega} \ell_{\omega} \left( \frac{w_{\omega}}{Y^*(\mathbf{w}, \underline{w})} K \left( \frac{Y^*(\mathbf{w}, \underline{w})}{e \underline{w}} \right) \right) d\omega.$$

Consequently, the corresponding  $\mathcal{V}(\mathbf{w}, \underline{w}) = \mathcal{V} \left( Y^*(\mathbf{w}, \underline{w}), \frac{Y^*(\mathbf{w}, \underline{w})}{\underline{w}} \right)$  is unique as well.

## C [Online Appendix] Quantitative illustration

In this section, I illustrate how Theorem (25) can be used to characterize the equilibrium reservation wage, labor allocation, and welfare level for any vector of wage offers  $\mathbf{w}$  and  $\underline{w}$ .

**Parameterization and Wage-Offer Distribution.** The baseline model sets  $\sigma = 0.035$ ,  $\chi = 0.3$ , and an individual contact rate  $\lambda_{\omega}$  such that  $M\lambda_{\omega} = 0.5$  for all  $\omega$ , where  $M$  denotes the number of firms. In the simulations reported,  $M = 3000$  and hence  $\lambda = 0.5/5000$ .

Wage offers  $\{w_i\}_{i=1}^M$  are drawn from a lognormal distribution  $\text{Lognormal}(\mu, \log \sigma_w)$  for  $\mu = 0.5$  (the “log\_mean”) and  $\log \sigma_w = 0.4$ , unless otherwise indicated. Specifically, each wage  $w_i$  is generated by

$$w_i = \exp(\mu + \log \sigma_w \cdot z_i),$$

where  $z_i \sim N(0, 1)$  is an independent standard normal draw. After simulating these  $M$  wages, the distribution is shifted upward so that its minimum equals the chosen  $\underline{w}$ . Formally,

$$w_i \leftarrow w_i - \min_j w_j + \underline{w},$$

then the resulting sample  $\{w_1, \dots, w_M\}$  is sorted in ascending order for use in the fixed-point routine.

**Fixed Point Routine** Given a vector of wage offers  $\mathbf{w}$  and the value of non-employment  $\underline{w}$ , I solve for the mean wage of employed workers  $W$  and the total employment income  $Y$  that solves the fixed point in equations (24)-(25). Computing  $L(\frac{Y}{eW})$  and  $K(\frac{Y}{eW})$  using equation 23 and plugging into 21 then yields  $\mathcal{V}(\mathbf{w}, \underline{w})$ . Finally, the distribution of employment across firms is computed as  $\mathbf{n}(\mathbf{w}, \underline{w}) =$



$$\left\{ n_i = \ell \left( \frac{w_i}{Y} K \left( \frac{Y}{e\bar{w}} \right) \right) \right\}_{i=1}^M.$$

**Results** Figure 3 displays the quantitative results. The top panels display key equilibrium outcomes: the ratio of welfare to earnings ( $\mathcal{V}/Y$ ) and the mean-min wage ratio ( $W/w_r$ ) across different parameter values. In the left panels, the value of non-employment ( $\underline{w}$ ) varies from 0.1 to 0.5, while in the right panels, the wage distribution shifts with the mean wage offer increasing from 1.8 to 2.6.

The top-left panel shows that as the value of non-employment increases, welfare relative to earnings rises while the mean-min wage ratio declines. Hence, an improvement in the outside option allows workers to become more demanding, reducing wage inequality among employed workers and increasing the contribution of non-employment to overall welfare. Conversely, the top-right panel demonstrates that higher and more dispersed wage offers intuitively increase the relative importance of employment for total welfare, while increasing wage inequality.

The bottom panels present the corresponding firm-level employment distributions for different parameterizations. These cumulative distribution functions display the characteristic rightward skew of employment predicted by job ladder models, with higher-paying firms capturing disproportionately more workers. The employment distributions shift rightward as either the value of non-employment increases (bottom-left) or the mean of the log-normal wage offer distribution rises (bottom-right), demonstrating how changes in either parameter affect the equilibrium allocation of workers across firms with different wage offers.

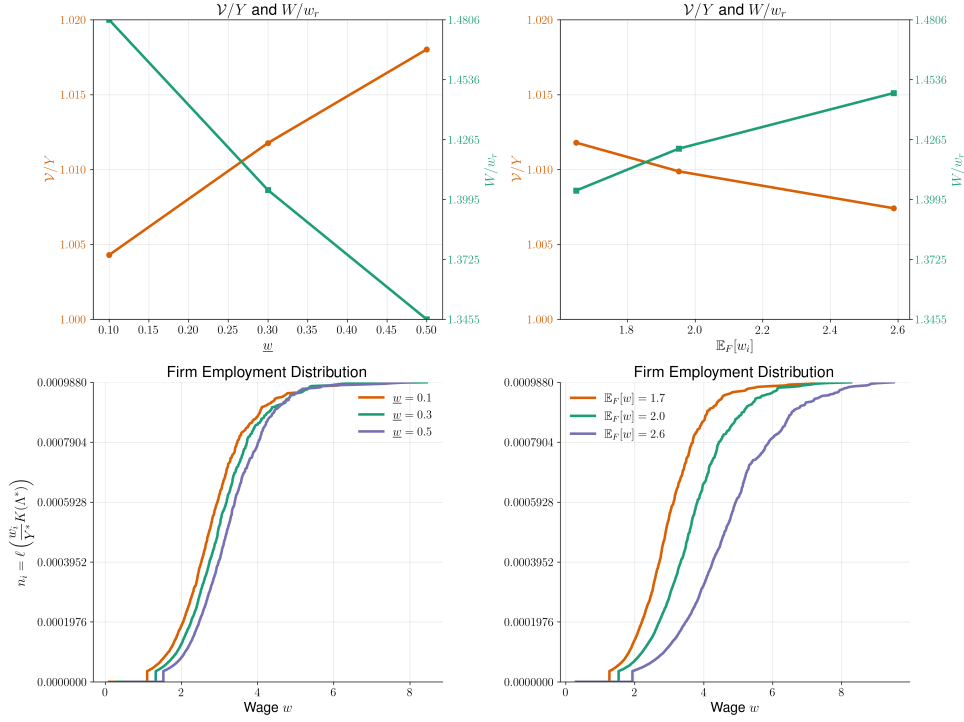


Figure 3: Quantitative Illustration

*Note:* The top panels display the ratios of welfare to earnings  $V/Y$  and of the mean wage to the reservation wage implied by Theorem 1 under a given vector of wage offers  $\mathbf{w}$  with cardinality  $|\Omega| = 5000$ , and the bottom panels shows the corresponding firm-level employment distribution attained from Corollary 1. All experiments parameterize the search technology at  $\sigma = 0.035$ ,  $\lambda_i = 0.5/5000$  and  $\chi = 0.3$ . The experiments in the left-side graphs vary  $\bar{w}$  from 0.1 to 0.5 while drawing wage offers from from the same lognormal distribution,  $w \sim \log \mathcal{N}(\mu = 0.5, \sigma = 0.3)$ . The right-side graphs set  $\bar{w} = 0.3$  and vary  $\mu$  from 0.5 to 1, with the resulting mean wage offer  $\mathbb{E}_F[w]$  displayed on the x-axis.