Heterogeneous Firms & Trade

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Topics

- **Q:** What are the effects of trade liberalization on welfare and productivity**?**
- Plan: Two key models of underlying much (if not, most) recent work in trade
  1. Melitz (2003): Increasing returns and monopolistic competition
  2. Eaton-Kortum (2002): Constant returns and perfect competition

Melitz 2003

Introduction

- Mounting evidence on heterogeneity of firms within sectors
  - Productivity, skill composition, wages, trade participation, organization
- Melitz JMP has become the workhorse model to analyze the effects of trade liberalization in the presence of firm heterogeneity
- Key channel: Selection effects of trade liberalization
  - Raised import competition displaces the worst (least productive firms), while firms that export expand and take advantage of cheaper foreign market access.
  - This reallocation raises aggregate productivity and welfare

Motivation (Bernard et al 2007)

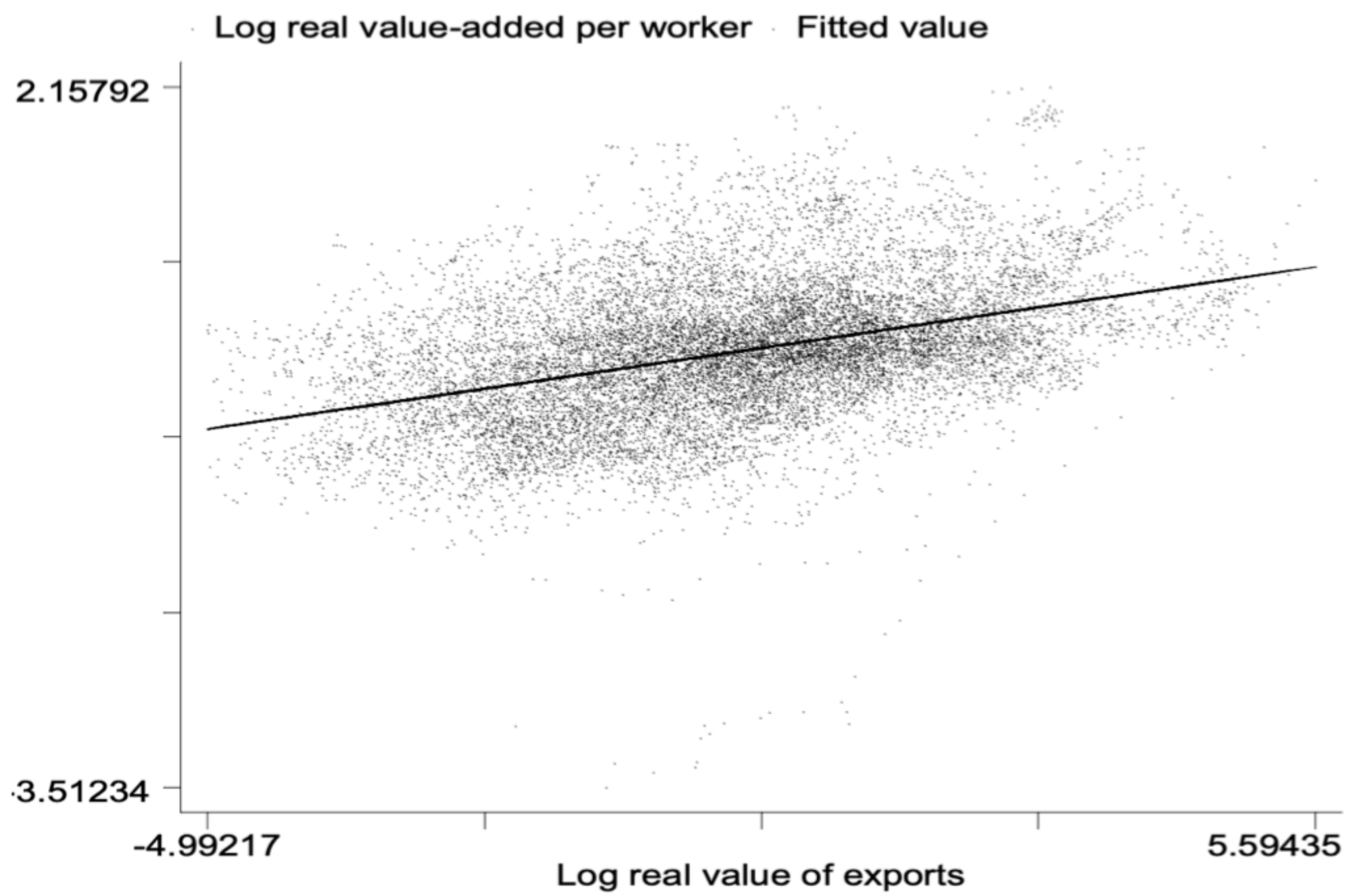
Exporting By U.S. Manufacturing Firms, 2002

NAICS industry	*Percent of firms*	*Percent of firms that export*	*Mean exports as a percent of total shipments*
311 Food Manufacturing	6.8	12	15
312 Beverage and Tobacco Product	0.7	23	7
313 Textile Mills	1.0	25	13
314 Textile Product Mills	1.9	12	12
315 Apparel Manufacturing	3.2	8	14
316 Leather and Allied Product	0.4	24	13
321 Wood Product Manufacturing	5.5	8	19
322 Paper Manufacturing	1.4	24	9
323 Printing and Related Support	11.9	5	14
324 Petroleum and Coal Products	0.4	18	12
325 Chemical Manufacturing	3.1	36	14
326 Plastics and Rubber Products	4.4	28	10
327 Nonmetallic Mineral Product	4.0	9	12
331 Primary Metal Manufacturing	1.5	30	10
332 Fabricated Metal Product	19.9	14	12
333 Machinery Manufacturing	9.0	33	16
334 Computer and Electronic Product	4.5	38	21
335 Electrical Equipment, Appliance	1.7	38	13
336 Transportation Equipment	3.4	28	13
337 Furniture and Related Product	6.4	7	10
339 Miscellaneous Manufacturing	9.1	2	15
Aggregate manufacturing	**100**	**18**	**14**

Sources: Data are from the 2002 U.S. Census of Manufactures.

Motivation

Figure 1: Exports and Labor Productivity Levels in U.S. Manufacturing, 1958-1994 (year effects removed)



Motivation

Table 3

Exporter Premia in U.S. Manufacturing, 2002

	Exporter premia		
	(1)	*(2)*	*(3)*
Log employment	1.19	0.97	
Log shipments	1.48	1.08	0.08
Log value-added per worker	0.26	0.11	0.10
Log TFP	0.02	0.03	0.05
Log wage	0.17	0.06	0.06
Log capital per worker	0.32	0.12	0.04
Log skill per worker	0.19	0.11	0.19
Additional covariates	None	Industry fixed effects	Industry fixed effects, log employment

Coefficients from a regression: $\log y_f = \beta_y 1[f \text{ is exporter}] + FE + Contr$

Autarky setup

- One industry of production
- Firms
  - Differ in productivity φ, drawn from density $g(\varphi)$ with cdf $G(\varphi)$
  - Pay fixed costs f_e to draw a productivity (entry)
  - Then pay fixed costs f_d to start producing (overhead)
- Mass L of consumers
  - CES preferences with elasticity of substitution σ
  - Supply labor inelastically at wage w
- Market structure
  - Monopolistic competition
  - Free entry (no profits in equilibrium)

Consumers: Preferences and demand

- L identical consumers with CES preferences

$$u = \left[\int_{\Omega} q(\omega)^{(\sigma-1)/\sigma} d\omega \right]^{\sigma/(\sigma-1)}$$

- Letting Y denote total income, demand for variety is given by

$$q(\omega) = p(\omega)^{-\sigma} P^{\sigma-1} Y, \quad P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}$$

- Income comes from wages, paid to inelastically supplied labor: $Y = wL$

Firms

- Technology: A firm with productivity φ can produce q using l units of labor according to

$$l(\varphi) = f_d + \frac{q(\omega)}{\varphi(\omega)}$$

- Indexing firms by φ, profit-maximizing price of an active firm is a constant markup over marginal cost (CES + monopolistic competition)

$$p(\varphi) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$$

- Revenues:

$$R(\varphi) = \varphi^{\sigma-1}(\frac{\sigma}{\sigma - 1}\frac{w}{P})^{1-\sigma}wL$$

- CES-MP: Variable profits are a constant share $1/\sigma$ of revenues. Net profits:

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf_d \equiv \varphi^{\sigma-1}B - wf_d$$

Zero Profit Condition (ZPC)

- Zero-profit condition: A cutoff producer that is indifferent between serving the domestic market and not

$$\frac{1}{\sigma} r(\varphi^*) = w f_d \Rightarrow \Rightarrow \varphi_*^{\sigma-1} = w f_d / B$$

- Firms with draws $\varphi < \varphi_*$ do not participate in domestic market
- Selection: “Incumbents” more productive then entrants

- Note: Expression for φ^* contains price index
- A second condition is needed to pin down productivity threshold

Free entry condition (FE)

- Firms enter until expected operating profits upon entry equal zero

$$\int_{\varphi_*}^{\infty} \pi(\varphi) dG(\varphi) = wf_e$$

- Note: $\pi(\varphi) = (\varphi/\varphi_*)^{\sigma-1} B\varphi_*^{\sigma-1} - wf_d = [(\varphi/\varphi_*)^{\sigma-1} - 1]wf_d$
- This implies FE can be rewritten as:

$$f_e = f_d \int_{\varphi_*} [(\varphi/\varphi_*)^{\sigma-1} - 1] dG(\varphi) \equiv J(\varphi_*) f_d$$

- The function $J(\varphi_*)$ is monotonically decreasing
  - Intuition: Higher φ_* leads to more competition and lower av. profits
  - There exists a unique values φ_* that solves FE

The Mass of Firms

- Still need to solve for the mass of firms:
  - M_e firms pay to draw a productivity
  - M firms operate in equilibrium
- These two are related as follows:

$$M = M_e[1 - G(\varphi_*)]$$

Equilibrium

- Price index:

$$P^{1-\sigma} = M_e \int_{\varphi_*}^{\infty} (\frac{\sigma}{\sigma - 1} w)^{1-\sigma} \varphi^{\sigma-1} dG(\varphi)$$

- Labor market clearing

$$wL = M_e w f_e + M_e \int_{\varphi_*}^{\infty} \left[\frac{\sigma - 1}{\sigma} r(\varphi) + w f_d \right] dG(\varphi)$$

- Equilibrium: φ_*, P, M_e, w satisfying
    - Zero profit condition
    - Price index aggregation
    - Free entry
    - Labor Market Clearing

Solving for the Mass of firms and GE

- Normalize $w = 1$
- Already showed how to pin down φ_*
- Rewrite labor market condition using the same tricks:

$$L/M_e = \sigma f_d \int_{\varphi_*}^{\infty} \left(\frac{\varphi}{\varphi_*} \right)^{\sigma - 1} dG(\varphi)$$

- Pins down M_e

Trade Equilibrium

Introducing Trade

- Introduce a second symmetric country
  - Wages still normalized to 1, equal in both
- All varieties are differentiated, both within and across countries.
- To export, firms have to pay another fixed cost f_x
  - Why? To match the fact that only a subset of firms export
- Iceberg trade costs: ship $\tau > 1$ for 1 unit to arrive

Zero Cutoffs with Trade

- Domestic ZCP is still given as:

$$\frac{1}{\sigma} r_d(\varphi_d) = w f_d \Rightarrow \varphi_d^{\sigma-1} = \frac{f_d \sigma^\sigma}{L P_d^{\sigma-1} (\sigma - 1)^{\sigma-1}}$$

- Exporters prices and revenues

$$p_x(\varphi) = \frac{\sigma}{\sigma - 1} \tau / \varphi, \quad r_x(\varphi) = \left(\frac{\tau p(\omega)}{P^*} \right)^{1-\sigma} w^* L^*$$

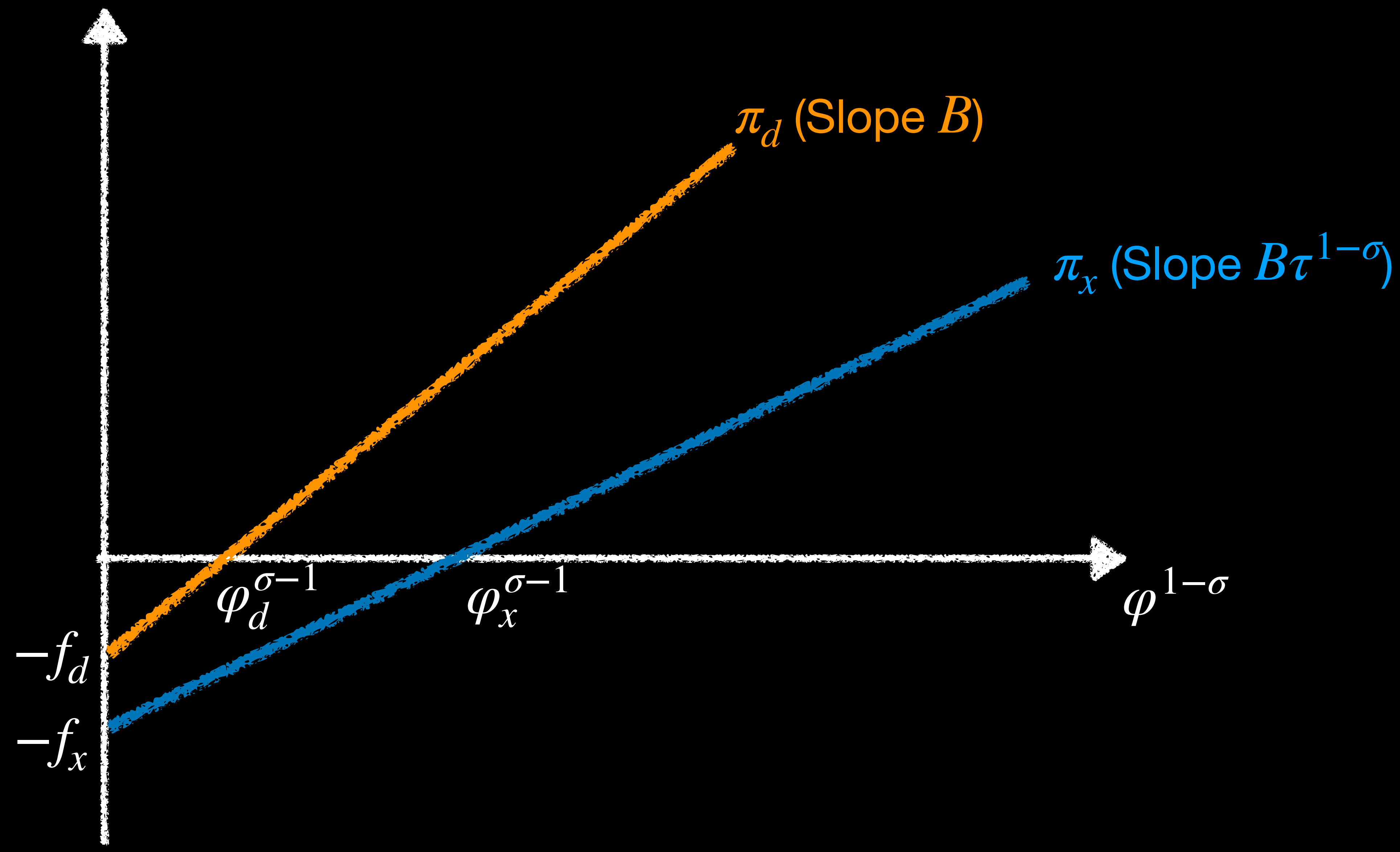
- Cutoff for exporting (imposing symmetry)

$$w f_x = \frac{1}{\sigma} r_x(\varphi_x) \Rightarrow \ \varphi_x^{\sigma-1} = \frac{f_x \tau^{\sigma-1} \sigma^\sigma}{L P_d^{\sigma-1} (\sigma - 1)^{\sigma-1}}$$

- Empirically relevant case: Selection into trade $(\varphi_x > \varphi_d)$ if $f_x \tau^{\sigma-1} > f_d$, since

$$(\varphi_x / \varphi_d)^{\sigma-1} = \tau^{\sigma-1} f_x / f_d$$

Graphical Representation



Free entry condition with exporting

- The free entry condition becomes:

$$
\begin{aligned}
f_e &= \int_0^\infty [\pi_d(\varphi) + \pi_x(\varphi)]dG(\varphi) \\
&= \int_{\varphi_d}^\infty [\frac{1}{\sigma} r_d(\varphi) - f_d]dG(\varphi) + \int_{\varphi_x}^\infty [\frac{1}{\sigma} r_x(\varphi) - f_x]dG(\varphi) \\
&= f_d \int_{\varphi_d}^\infty [(\varphi/\varphi_d)^{\sigma-1} - 1]dG(\varphi) + f_x \int_{\varphi_x}^\infty [(\varphi/\varphi_x)^{\sigma-1} - 1]dG(\varphi) \\
&\equiv J(\varphi_d) f_d + J(\varphi_x) f_x
\end{aligned}
$$

- $J(\,.\,)$ still monotonically decreasing
  - Since $\varphi_x > \varphi_d$, this implies that $\varphi_d > \varphi^*$ (Gains from trade!)
- Can show that trade cost τ “toughen” selection ($\varphi_d \uparrow$) by analyzing:

$$
f_e = \int_{\varphi_d}^\infty [(\varphi/\varphi_d)^{\sigma-1} - 1] f_d dG(\varphi) + \int_{\varphi_x(\varphi_d)}^\infty [(\varphi/\varphi_d)^{\sigma-1} \tau^{1-\sigma} f_d / f_x - 1] f_x dG(\varphi)
$$

Market Clearing

- Now accounts for export activity

$$wL = M_e w f_e + M_e \int_{\varphi_d}^{\infty} \left[\frac{\sigma - 1}{\sigma} r_d(\varphi) + w f_d \right] dG(\varphi) + M_e \int_{\varphi_x}^{\infty} \left[\frac{\sigma - 1}{\sigma} r_x(\varphi) + w f_x \right] dG(\varphi)$$

- Accounts for average labor used in domestic and export activity

Gains from Trade

- Welfare is simply inverse price index P
- CES price index:

$$P_d^{1-\sigma} = M_e \int_{\varphi_d}^{\infty} \tilde{\sigma} \varphi^{\sigma-1} dG(\varphi) + M_e^* \tau^{1-\sigma} \int_{\varphi_x^*}^{\infty} \tilde{\sigma} \varphi^{\sigma-1} dG(\varphi)$$

Gains from Trade

$$P_d^{1-\sigma} = M_e \int_{\varphi_d}^{\infty} \tilde{\sigma}\varphi^{\sigma-1} dG(\varphi) + M_e^* \tau^{1-\sigma} \int_{\varphi_x^*}^{\infty} \tilde{\sigma}\varphi^{\sigma-1} dG(\varphi)$$

- Three potential forces affecting gains from trade
    1. Second term only shows up with trade: Positive effect of increased import varieties on welfare
    2. As home opens to trade ($\tau \downarrow$), the least productive firms exit ($\varphi_d \uparrow$), inducing a selection effect, making the average product from home cheaper. Simultaneously the export cutoff falls, as more firms find it profitable to export (reallocation!)
    3. M_e falls as home opens to trade: Average revenues are higher per entrant, so less firms enter (this effect is due to the symmetry, in general it is ambiguous).

Sufficient Statistic for Welfare

- φ_d is a sufficient statistic for welfare from zero profit condition:

$$\frac{1}{\sigma} r_d(\varphi_d) = f_d$$

$$\Rightarrow \varphi^{\sigma-1} = \sigma \frac{f_d}{L} (\frac{\sigma - 1}{\sigma})^{1-\sigma} P^{1-\sigma}$$

$$\Rightarrow P^{1-\sigma} \propto \varphi_d^{\sigma-1}$$

- Trade liberalization raises cutoff, so it raises welfare.

What did we learn?

- Market-integration leads to reallocation of resources across firms within industries
  - Low productivity firms exit
  - Domestic sellers that survive contract
  - Exporting firms expand
- Sales weighted industry productivity rises due to this reallocation
- Missing: Selection does not feed back into changes in firm-level productivity

Applications

- The reallocation of resources between firms has been a key force highlighted by modern trade theory to study, e.g.,:
  - Wage inequality: More productive firms (i) pay higher wages and (ii) are more skill intensive
    - Reallocation shifts aggregate wage inequality and demand for skill
  - Innovation: Trade liberalization raises the size of export markets, scaling the returns to innovation. It also raises domestic competition, which has ambiguous effects on innovation
  - Markups: When markups vary (endogenously or exogenously) across firms, reallocation within industries upon trade liberalization affects aggregate markups
  - …
- Net effect: over 15k citations for Melitz

Eaton Kortum 2002

Introduction

- Another famous model with heterogenous firms
- Different from Melitz (2003) [IRS+MC] features CR + PC
- A probabilistic formulation of the canonical Dornbusch-Fisher-Samuelson model
  - Comparative advantage differences promote trade
  - Geographic barriers diminish trade
- The probabilistic formulation itself is a hugely influential technical contribution
  - Underlying technique also used in state-of-the-art quantitative models of migration

Model Set-up

- Countries are indexed by $i \in 1,\ldots,N$
- Continuum of goods $\omega \in [0,1]$
- Labor only factor of production
- Constant Returns to Scale production with $z_i(\omega)$ productivity of variety ω in country i
- Iceberg trade costs τ_{ij} from country i to j, where $\forall i, \tau_{ii} = 1$.

Preferences

- Consumers have CES preferences over the set of varieties $\omega \in [0,1]$
- Each variety is ω is homogeneous across countries
- Perfect competition, so prices equal marginal cost

$$p_{ij}(\omega) = \frac{w_i}{z_i(\omega)} \tau_{ij}$$

- Consumers in each country shop for the cheapest source country to buy each variety ω, so the price paid for ω in destination j equals

$$p_j(\omega) = \min_{i=1,\ldots,N} \{p_{ij}(\omega)\}$$

Technology

- Country $i's$ efficiency in producing variety ω is the realization of a random variable Z_i drawn from $F_i(z) = Pr(Z_i \leq z)$
  - By LLN $F_i(z)$ is fraction of varieties for which country i has efficiency below z
- Eaton-Kortum choose F to be the Frechet distribution

$$F_i(z) = exp(T_i z^{-\theta}), T_i > 0$$

  - T_i captures absolute comparative advantage of country i
  - θ is a (inverse) measure of the degree of comparative advantage

Key Property of Extreme Value Distributions

- Distributions in the class of extreme value distributions are “max and min stable”
  - E.g.: Frechet, Gumbel, multivariate versions of those
- In other words: The minimum or maximum of a list of i.i.d. Frechet variables follows a freshet distribution

$X_{min} = \min\{x_1, x_2, \ldots, x_n\}$ and $x_i \sim$ Frechet then $X_{min} \sim$ Frechet

- This property is very useful to economists
- Only extreme value distributions are “max and min stable”

Prices

- The origin country i presents a destination j with a distribution of prices $G_{ij}(p) = Pr[p_{ij} \leq p] = 1 - F_i(w_i\tau_{ij}/p)$:

$$G_{ij}(p) = 1 - \exp\left(-\left[T_i(w_i\tau_i j)^{-\sigma}\right] p^{-\sigma}\right)$$

- The distribution of the minimum of prices (i.e. the actual price paid by consumers) in destination j for any variety is

$$G_j(p) = Pr(p_j \leq p) = 1 - \prod_{i=1}^{N} [1 - G_{ij}(p)]$$

- Substituting yields:

$$G_j(p_) = 1 - \exp(-\Theta_j p^\theta) \text{ where } \Theta_j \equiv \sum_i T_i(w_i\tau_{ij})^{-\theta}$$

Corollaries of the Frechet assumption

- The probability that country i provides a given good at the lowest price:

$$Pr[p_{ij} \leq \min_{k \neq i} p_{kj}] \equiv \pi_{ij} = \frac{T_i(w_i \tau_i j)^{-\theta}}{\Theta_j} = \frac{T_i(w_i \tau_i j)^{-\theta}}{\sum_i T_i(w_i \tau_{ij})^{-\theta}}$$

  - LLN: this is also the fraction of goods j purchases from i

- Frechet property: The price of a good that country n actually buys from any country i also has the distribution $G_i(p)$
  - So paid prices from any origin are the same (conditional on the set of goods that a given origin provides)…

Corollaries of the Frechet assumption

- With CES utility and a Frechet distribution for the prices actually paid, the price index takes the following form:

$$P_j = \Gamma \Theta_j^{-1/\Theta} \text{ where } \Gamma \equiv \gamma(\frac{\theta + 1 - \sigma}{\theta})^{1/(1-\sigma)}$$

- Θ_j summarizes technology, input costs, and geographic barriers around the world
  - i.e., captures country's access to consumables

Trade flows and Gravity

- Recall that distribution of prices actually paid in i for goods from j equals G_i
- So the fraction of goods sourced from any origin is also the fraction of total spending spent on that origin!
  - Since average expenditure per good does not vary across origins
- Total volume of production in country i

$$Q_i = \sum_j X_{ij} = \sum_j \pi_{ij} X_j = T_i w_i^{-\theta} \sum_j \frac{\tau_{ij}^{-\theta} X_j}{\Theta_j} = T_i w_i^{-\theta} \gamma \sum_j X_j \left(\frac{\tau_{ij}}{P_j} \right)^{-\theta}$$

$\equiv \Pi_i$

$$X_{ij} = \pi_{ij} X_j = \frac{T_i w_i^{-\theta} \tau_{ij}^{-\theta} \gamma}{P_j^{-\theta}} X_j$$

- Solving for $T_i w_i^{-\theta}$ in the first, and substituting into the second equation yields the gravity equation

$$X_{ij} = \tau_{ij}^{-\theta} \left(\frac{Q_i}{\Pi_i^{-\theta}} \right) \left(\frac{X_j}{P_j^{-\theta}} \right)$$

Gravity

$$X_{ij} = \tau_{ij}^{-\theta} \left(\frac{Q_i}{\Pi_i^{-\theta}} \right) \left(\frac{X_j}{P_j^{-\theta}} \right)$$

- Interpretation: Π_i captures market access by producers, P_j captures market access by consumers
- Taking logs yields:

$$\ln X_{ij} = -\theta \ln \tau_{ij} + \ln Q_i + \ln X_j + \theta(\ln \Pi_i + P_j)$$

- In this simple model (absent intermediates), sales=expenditures=GDP.
    - Then the first three terms correspond to the traditional “gravity” equation
- The last term captures “multilateral resistance”: An error term in RF gravity
    - Trade not only depends on bilateral resistance, but also on the importers access to consumables and the exporters access to consumers

Equilibrium

- All endogenous objects can be expressed as a function of $\{w_i\}_i$
- Goods market provides a systems of N equations in N variables

$$w_i L_i = \sum_j \pi_{ij} w_j L_j$$

- Simple iterative procedure can solve efficiently for w

1. Guess wages
2. Compute trade shares π_{ij} given the guess
3. Compute new wages implied by the equation above.
4. Compare to initial guess, update and repeat.

Welfare

- Rearranging π_{nn} we obtain:

$$\pi_{ii} = \Gamma \frac{T_i w_i^{-\theta}}{P_i^{-\theta}} \Rightarrow \frac{w_i}{P_i} = \Gamma(\frac{T_i}{\pi_{ii}})^{\frac{1}{\theta}}$$

- So gains from trade show up in own-trade share
  - Reflects revealed preference: "How much am I borrowing abroad's technology?"
  - Gains greater the more heterogeneity there is in efficiency.
- π_{nn} and θ are sufficient to calculate welfare changes in response to changes in fundamentals in any other country and any trade cost.
  - E.g., going from baseline (1990) to autarky $\pi_{nn} = 1$ implies losses between -0.2% and -10% (smallest for Japan and US (-0.8%).
  - "Missing" gains from trade?

Welfare more generally

- Assuming Pareto distribution $G(\varphi) = 1 - (\varphi_{\min}/\varphi)^{\theta}$ in Melitz (2003), we can show that

$$\frac{w_i}{P_i} \propto \left(\frac{1}{\pi_{ii}}\right)^{1/\theta}$$

- The same sufficient statistic for welfare!
- Arkolakis, Costinot, and Rodriguez-Clare (2012) provide sufficient conditions for a generalization of these patterns
   1. Balanced trade $\sum_i X_{ij} = \sum_j X_{ij}$
   2. Profits are constant share of revenue (holds for perfect competition or CES + MC)
   3. CES import demand: $-\dfrac{\partial \ln(X_{ij}/X_{jj})}{\partial \ln \tau_{i'j}} = \epsilon$ if $i' = i$ and 0 otw
- Then: changes in welfare can be computed: is $d \log W_i = -\dfrac{1}{\epsilon} d \ln \pi_{ii}$

Predicting welfare effects

- Welfare effect of a counterfactual change in other-country fundamentals entirely predicted and identical, up to ϵ
- Of course, structurally
  - Parameters (like trade elasticity) may have different interpretations
  - Mechanisms for welfare change may be different
- Main question for any trade model: How do you depart from or add to ACR 2012?
- Arkolakis, Costinot, Donaldson, Rodriguez-Clare conduct similar analysis but in variable markup setting